

DFG-Research Unit

“Earth rotation and Global Dynamic processes”

Comparison and Combination of
SLR Solutions
Including Gravity Field Coefficients
and Range Biases

N. Panafidina, M. Rothacher, D. Thaller



Poznan, 13 – 17 October 2008

Used dataset

GGOS-D: consistent modelling

Weekly solutions in SINEX files, 1993 – 2007

SLR GFZ:

Station coordinates

ERP with 24h resolution

Range biases for some stations

Gravity field coefficients up to degree and order 2

SLR DGFI:

Coordinates, 24h ERP, Range biases

Gravity field coefficients separately for Lageos1 & Lageos2

Solutions

- The influence of range biases on the solution
- The influence of estimating gravity field coefficients

Weekly solutions:

Only station coordinates and ERP estimated

Gravity field coefficients fixed

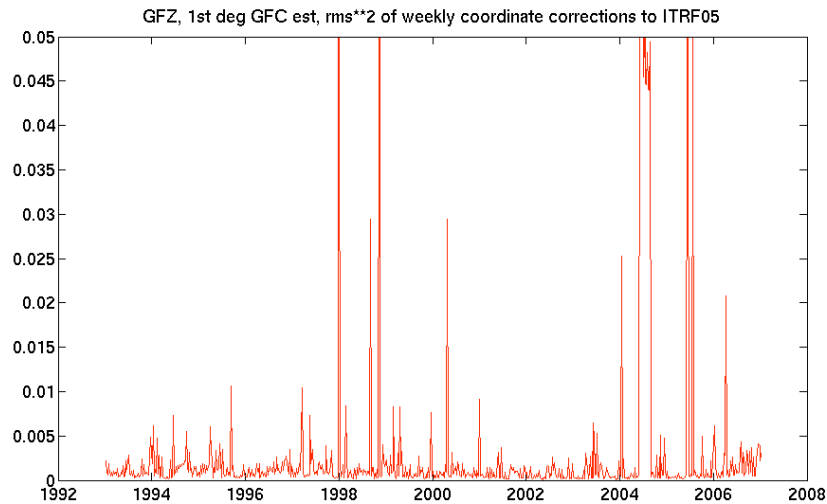
NNR + 1st UT1 fixed

stations with large deviations for
a certain week not used for datum

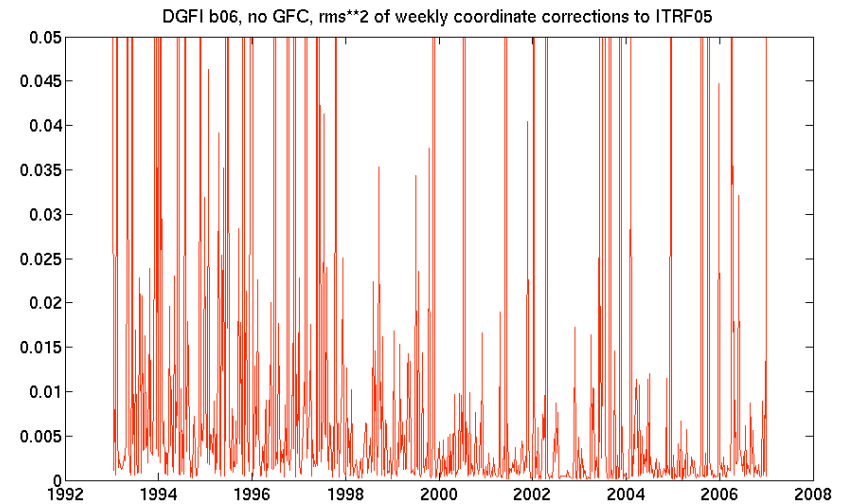
DGFI & GFZ solutions: coordinates

Weekly squared RMS of coordinate corrections

GFZ

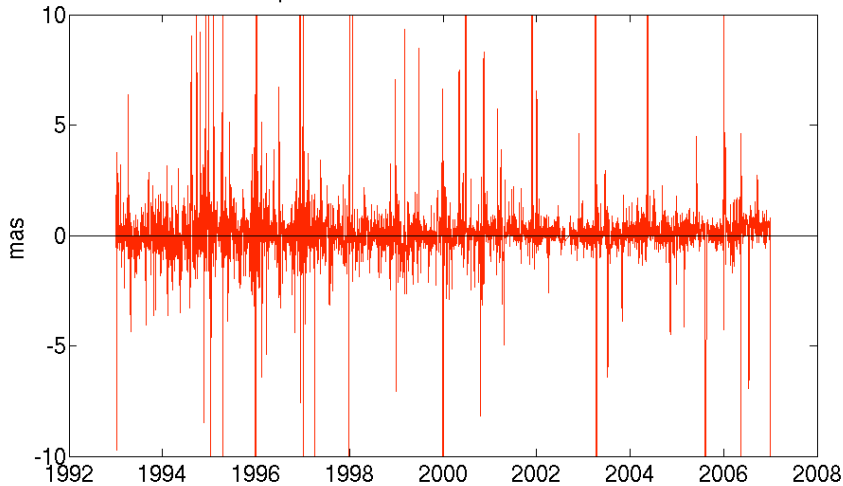


DGFI

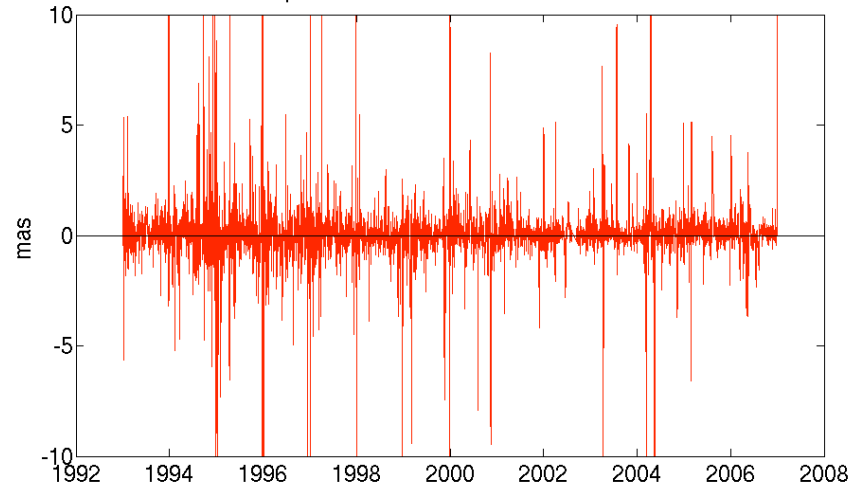


DGFI SLR solution: ERP w.r.t. C04

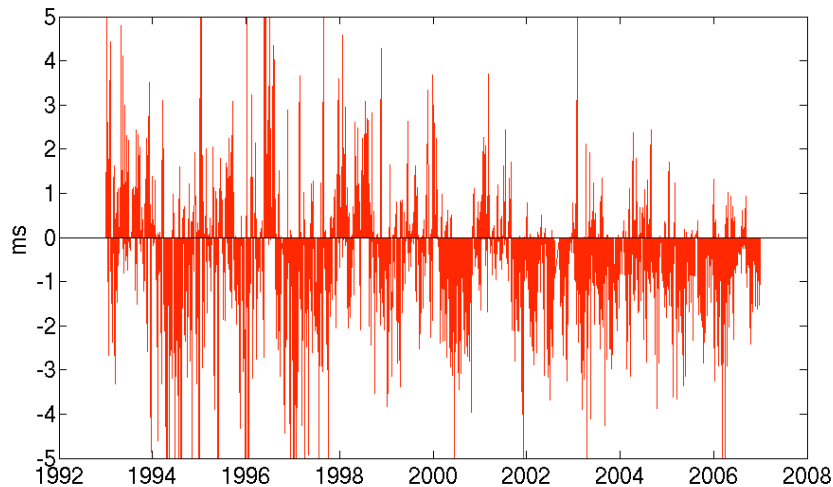
X pole: DGFI GGOS-D w.r.t. C04



Y pole: DGFI GGOS-D w.r.t. C04



UT1: DGFI GGOS-D w.r.t. C04



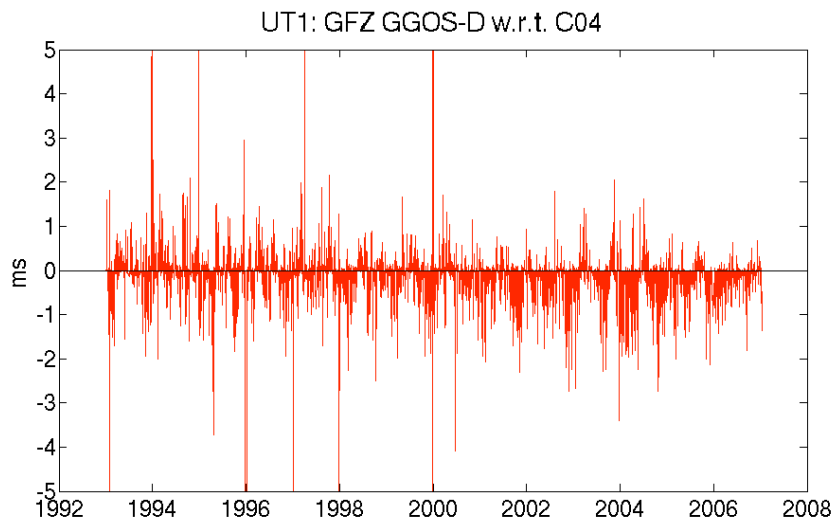
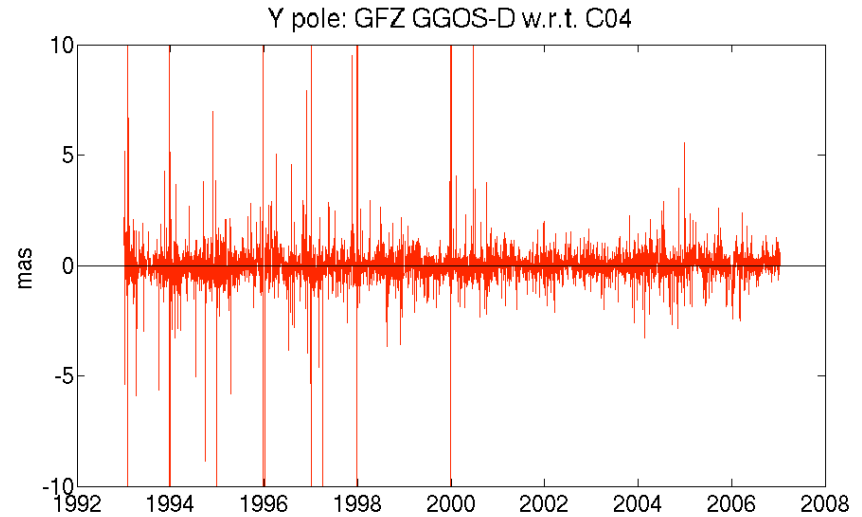
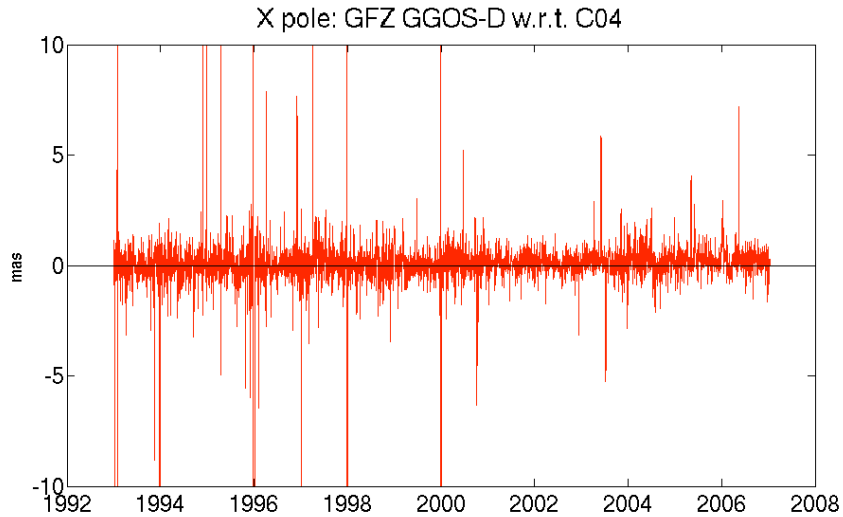
WRMS (outliers removed)

X 0.39 mas

Y 0.40 mas

UT1 0.00012 ms

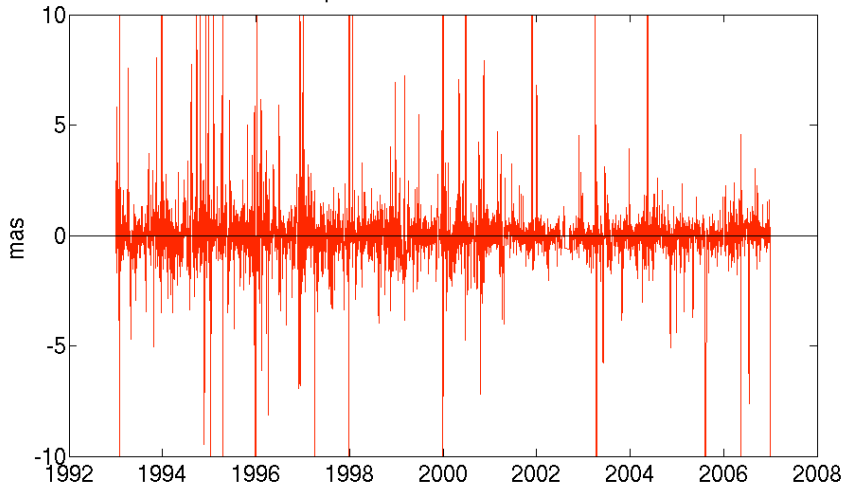
GFZ SLR solution: ERP w.r.t. C04



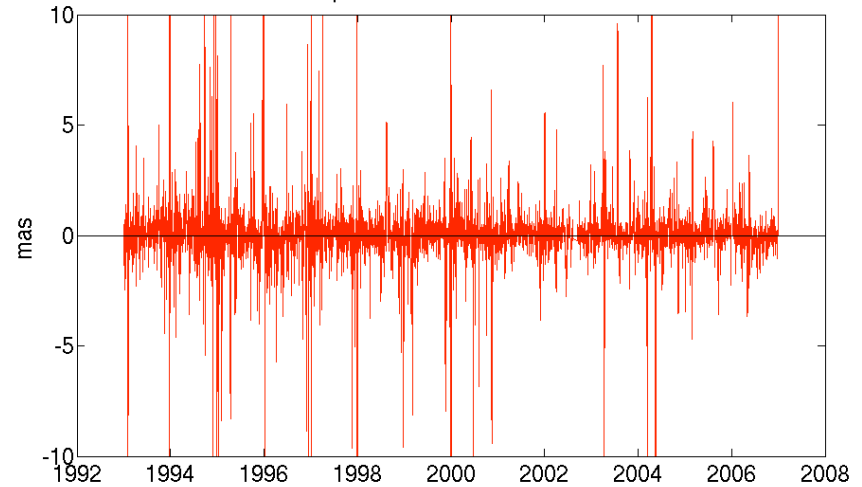
	WRMS (outliers removed)
X	0.35 mas
Y	0.35 mas
UT1	0.00011 ms

GFZ & DGFI solutions: ERP differences

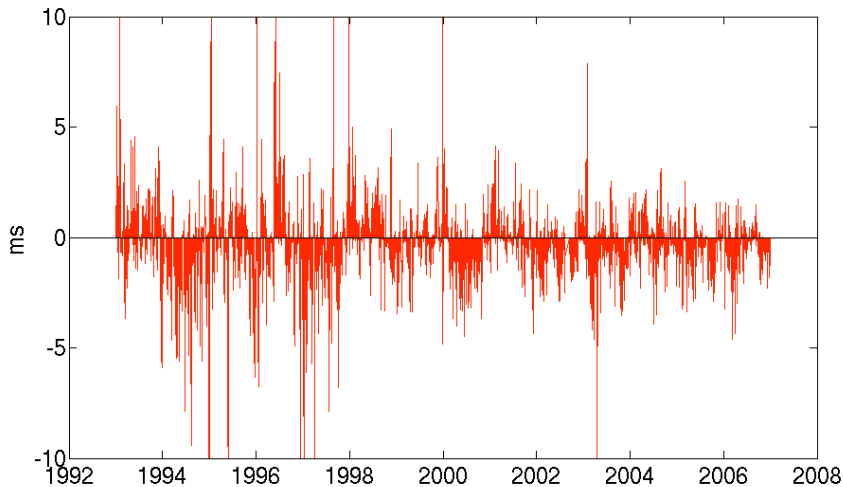
X pole: DGFI minus GFZ



Y pole: DGFI minus GFZ



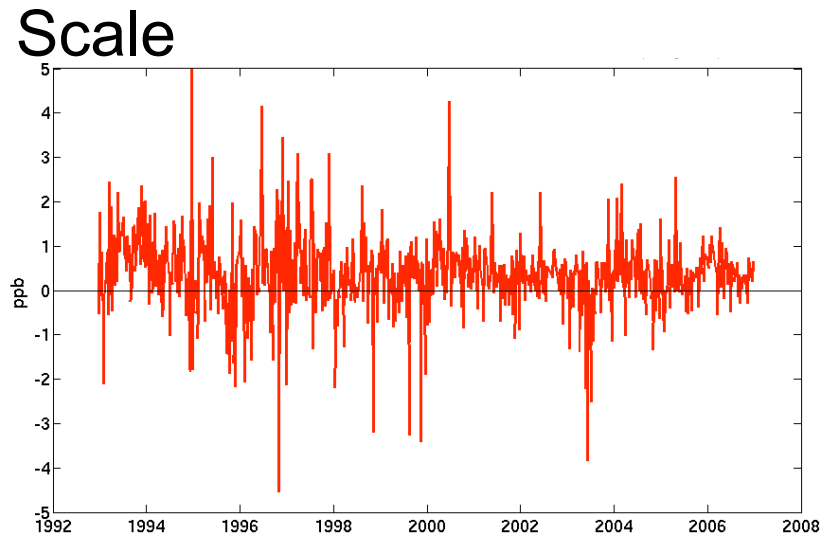
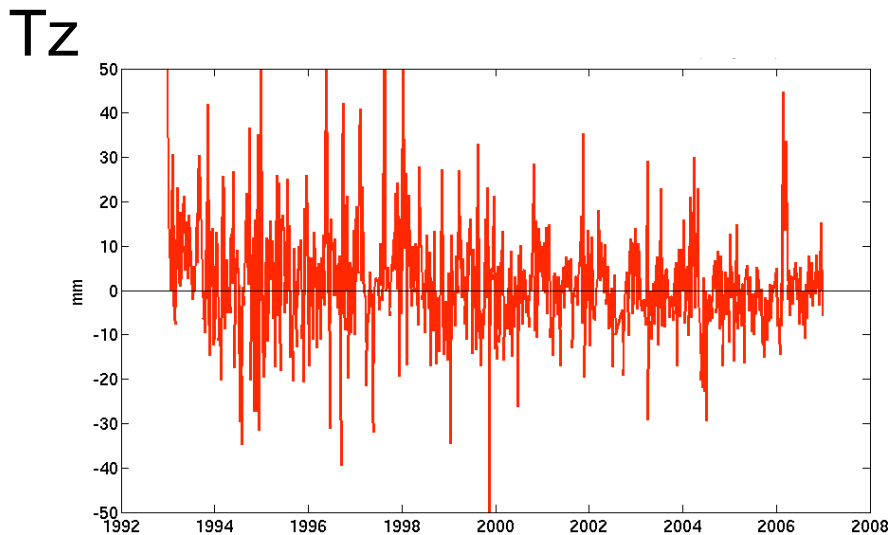
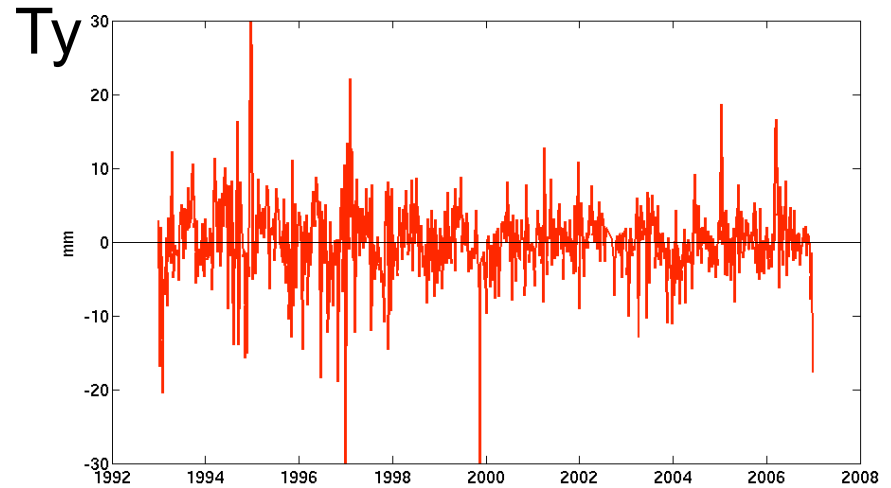
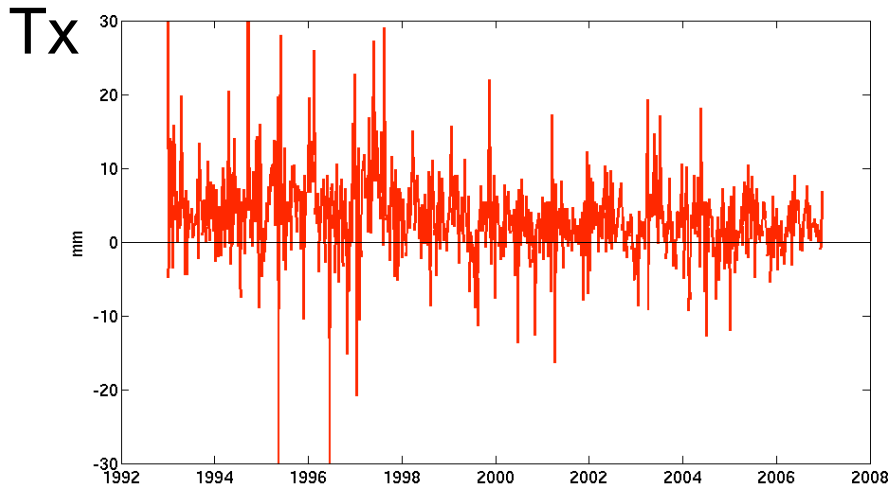
UT1: DGFI minus GFZ



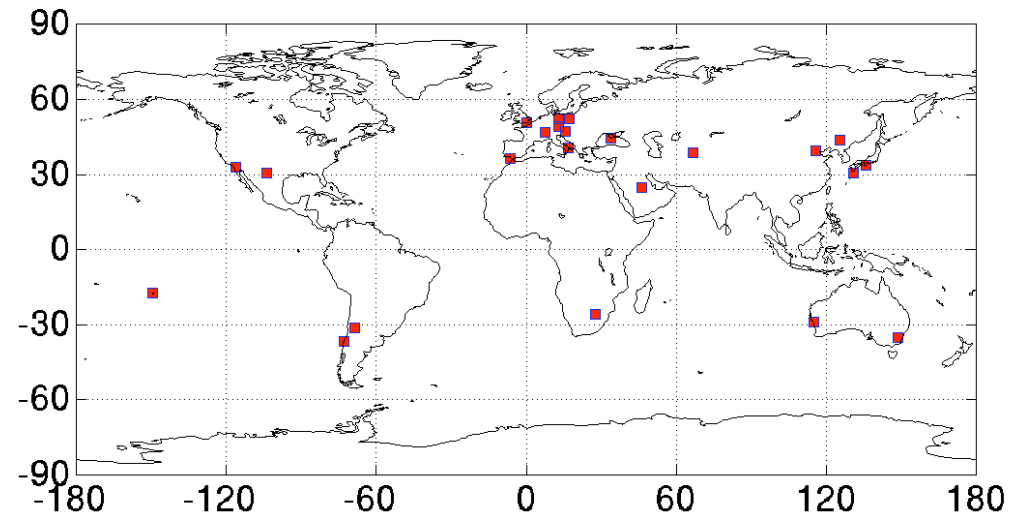
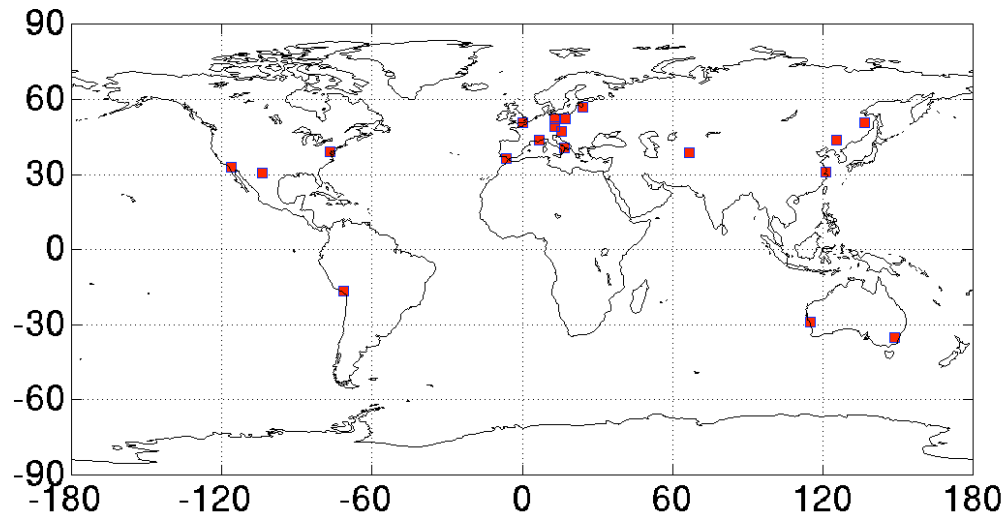
No systematic difference
in ERPs

DGFI & GFZ solutions: coordinates

Helmert parameters between DGFI and GFZ solutions



Network geometry



Network geometry: correlations

Helmert parameters:

$$\vec{X}_{\text{est}} = \mu \cdot \mathbf{R} \cdot \vec{X}_{\text{ref}} + \vec{T}$$

μ scale factor

\mathbf{R} matrix of rotations

\mathbf{T} translations

Design matrix:

$$\mathbf{A} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 0 & 0 & x_i & 0 & z_i & -y_i \\ 0 & 1 & 0 & y_i & -z_i & 0 & x_i \\ 0 & 0 & 1 & z_i & y_i & -x_i & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

Normal equation matrix:

$$\begin{pmatrix} N & 0 & 0 & \sum x_i & 0 & \sum z_i & -\sum y_i \\ 0 & N & 0 & \sum y_i & -\sum z_i & 0 & \sum x_i \\ 0 & 0 & N & \sum z_i & \sum y_i & -\sum x_i & 0 \\ \sum x_i & \sum y_i & \sum z_i & \sum r_i^2 & 0 & 0 & 0 \\ 0 & -\sum z_i & -\sum y_i & 0 & \sum z_i^2 + y_i^2 & -\sum x_i y_i & -\sum x_i z_i \\ \sum z_i & 0 & -\sum x_i & 0 & -\sum x_i y_i & \sum z_i^2 + x_i^2 & -\sum y_i z_i \\ -\sum y_i & \sum x_i & 0 & 0 & -\sum x_i z_i & -\sum y_i z_i & \sum y_i^2 + x_i^2 \end{pmatrix}$$

Network geometry: correlations

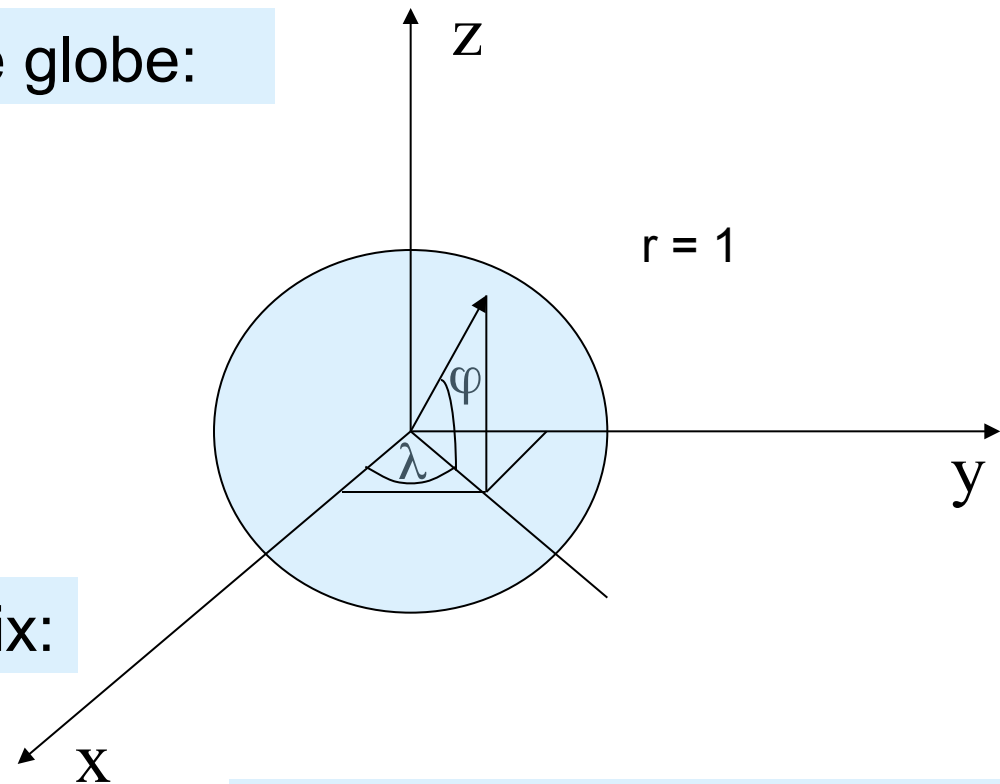
Uniform distribution over the globe:

$$\int_{-\pi/2}^{\pi/2} \int_0^{2\pi} d\varphi d\lambda \cos \varphi$$

$$x = \cos \varphi \cos \lambda$$

$$y = \cos \varphi \sin \lambda$$

$$z = \sin \varphi \longrightarrow 2\pi \frac{\sin^2 \varphi}{2} \Big|_{\varphi_1}^{\varphi_2}$$

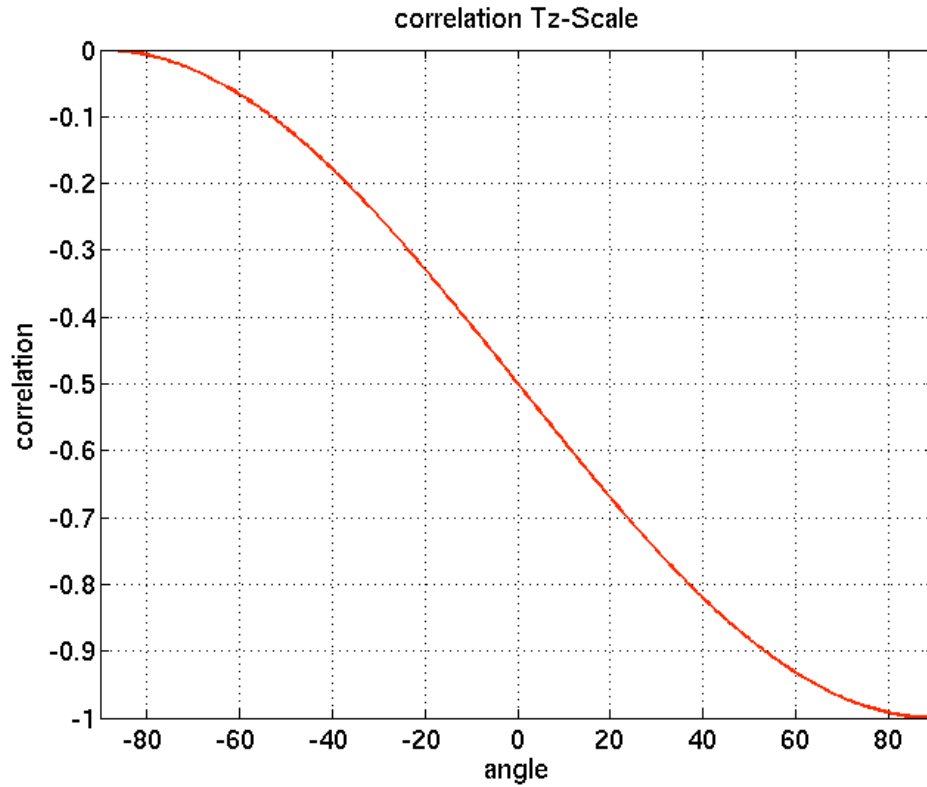


Covariance/correlation matrix:

	Tx	Ty	Tz	Sc	Rx	Ry	Rz
Tx	A	0	0	0	0	B	0
Ty	0	A	0	0	-B	0	0
Tz	0	0	A	B	0	0	0
Sc	0	0	B	A	0	0	0
Rx	0	-B	0	0	A ₁	0	0
Ry	B	0	0	0	0	A ₂	0
Rz	0	0	0	0	0	0	A ₃

Asymmetric distribution over latitude cause correlations Tx - Ry, Ty - Rx and Tz - Scale

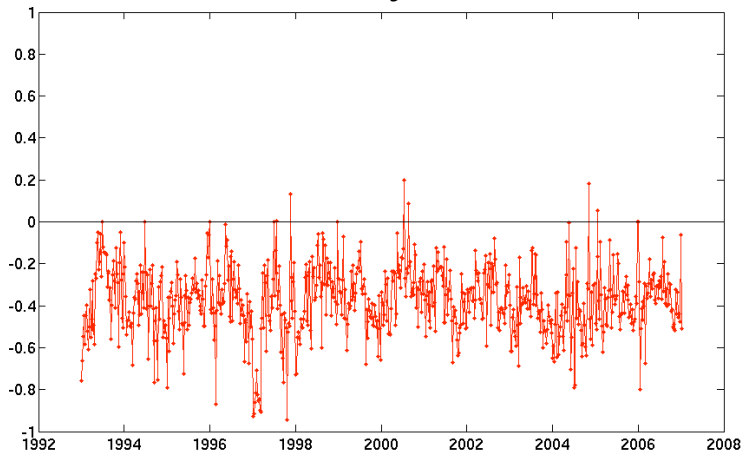
Network geometry: correlations



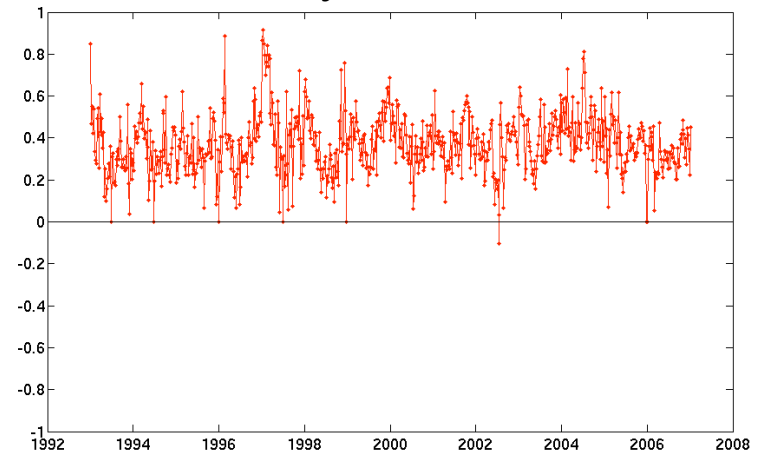
$$-\frac{1}{2}(\sin \varphi + 1)$$

Network geometry: correlations

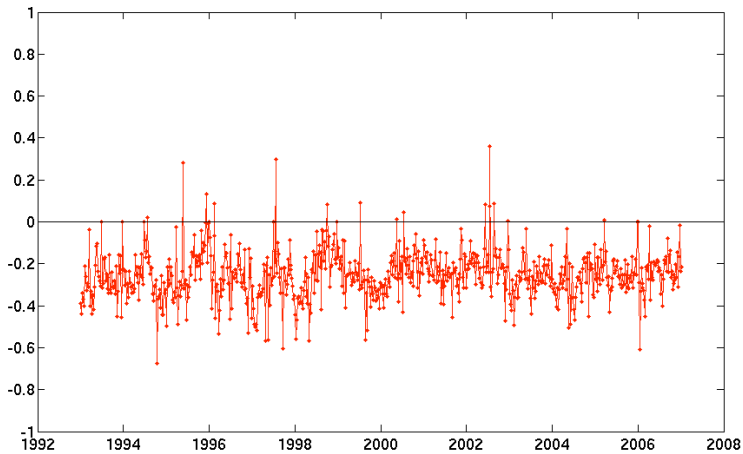
Tx-Ry



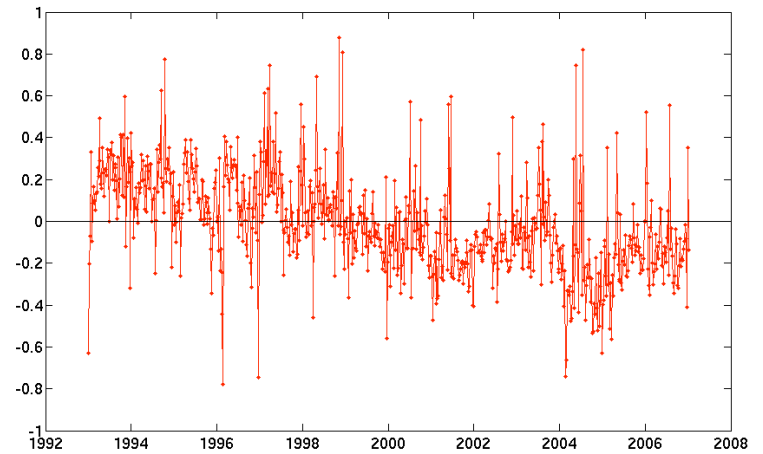
Ty-Rx



Tz-Scale



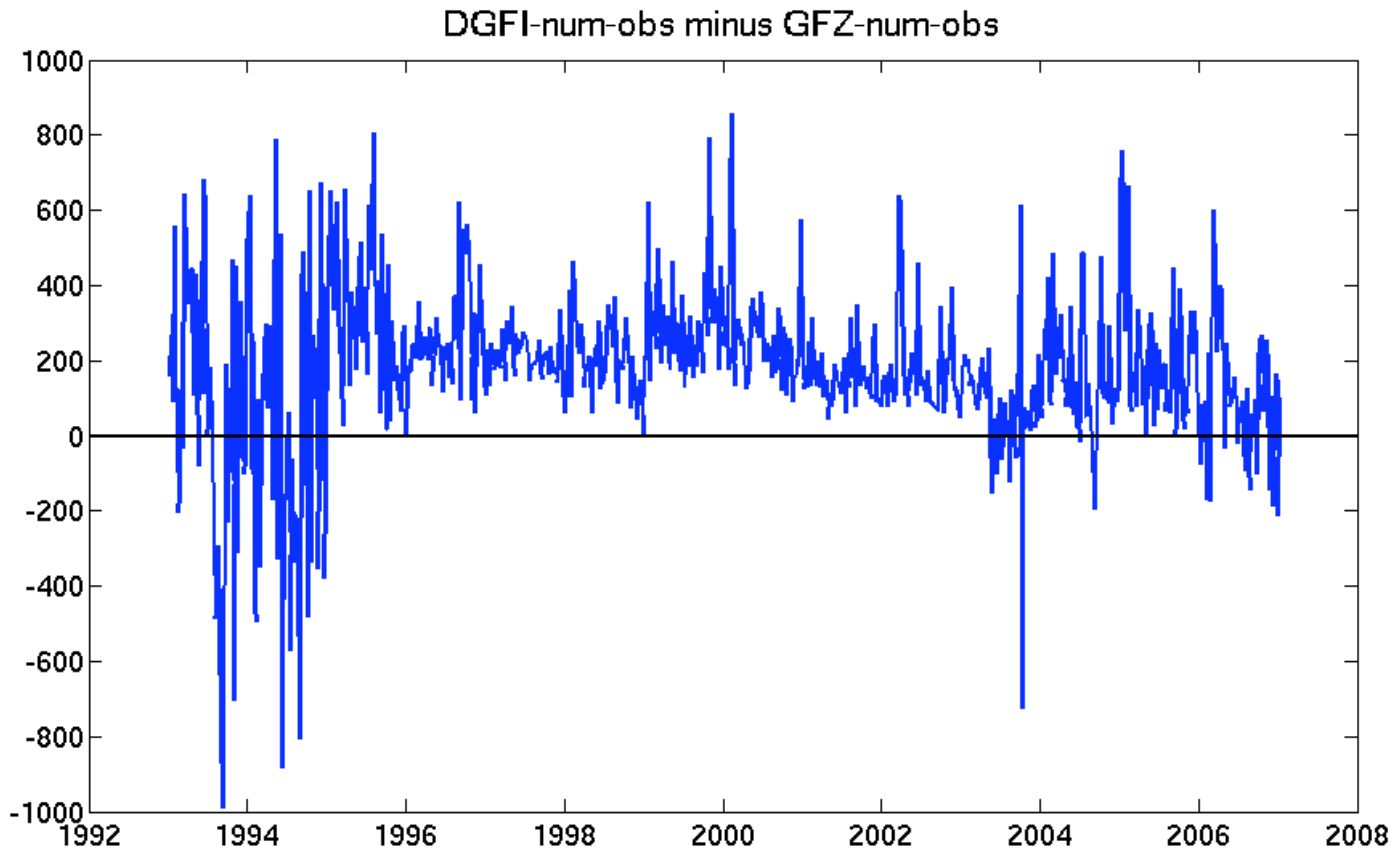
Tz-Rx



DGFI & GFZ SLR solutions

Number of observations

DGFI has ~200 observations more



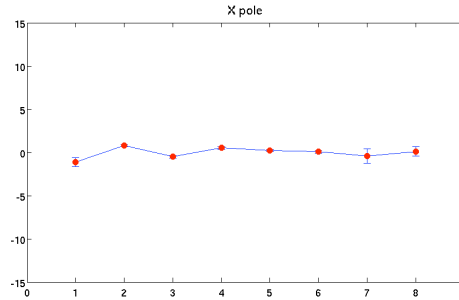
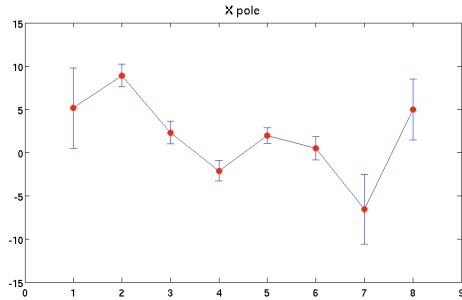
DGFI SLR solution: bad observations?

DGFI

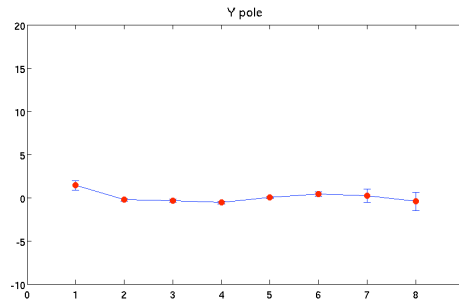
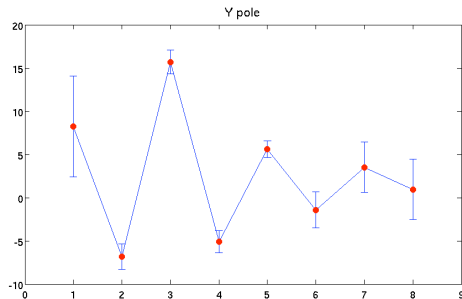
GFZ

Week 98 354

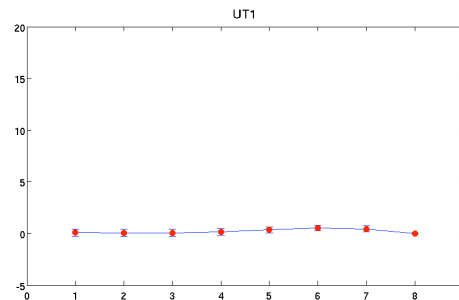
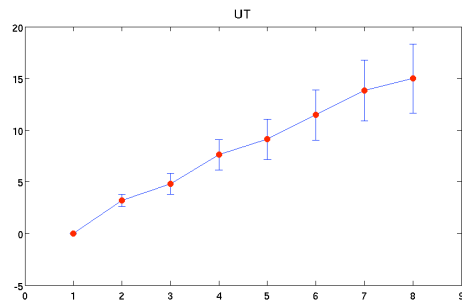
X



Y



UT



Station 7811 Borowiec is not in GFZ solution

In DGFI solution correction to a priori: ~ 4 m

Number of observations
GFZ: 1252
DGFI: 1370

GFZ SLR solution: lack of observations?

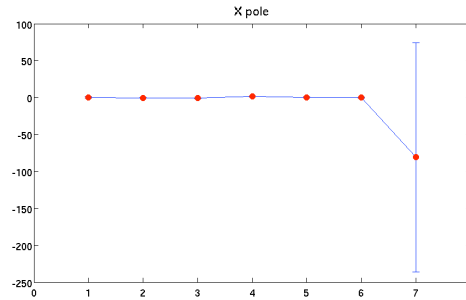
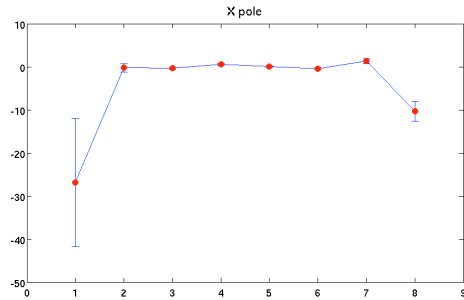
DGFI

GFZ

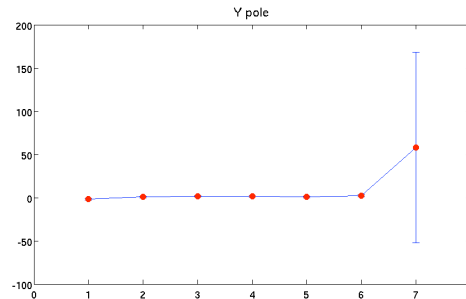
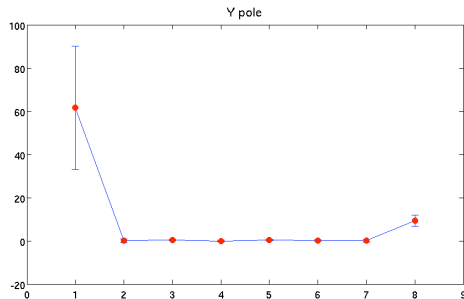
Week 99 360

GFZ: singular ERP

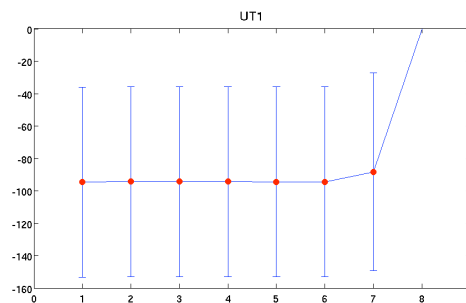
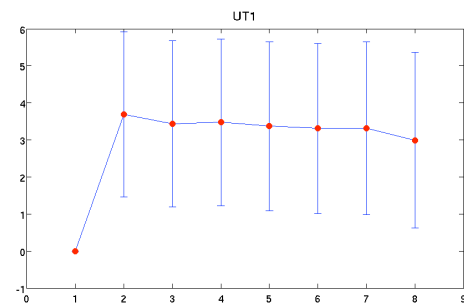
X



Y



UT



Number of observations

GFZ: 770

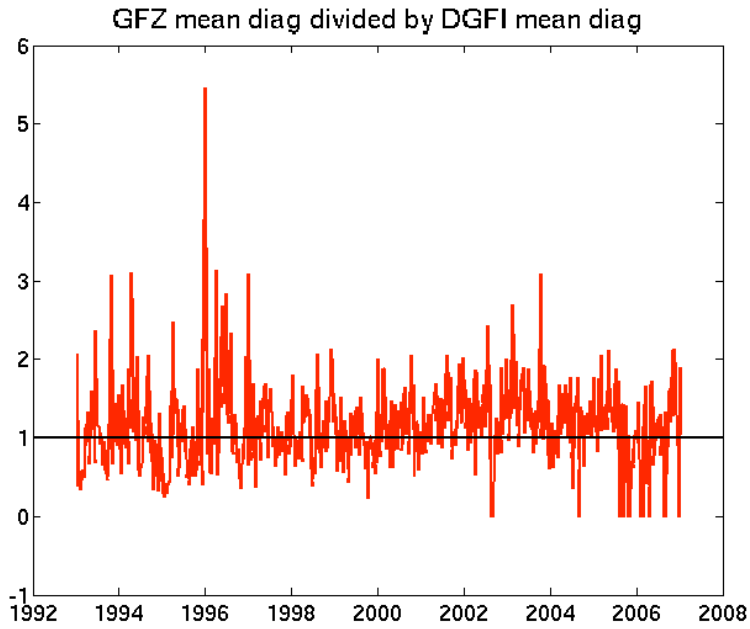
DGFI: 1113

Number of stations

GFZ: 12

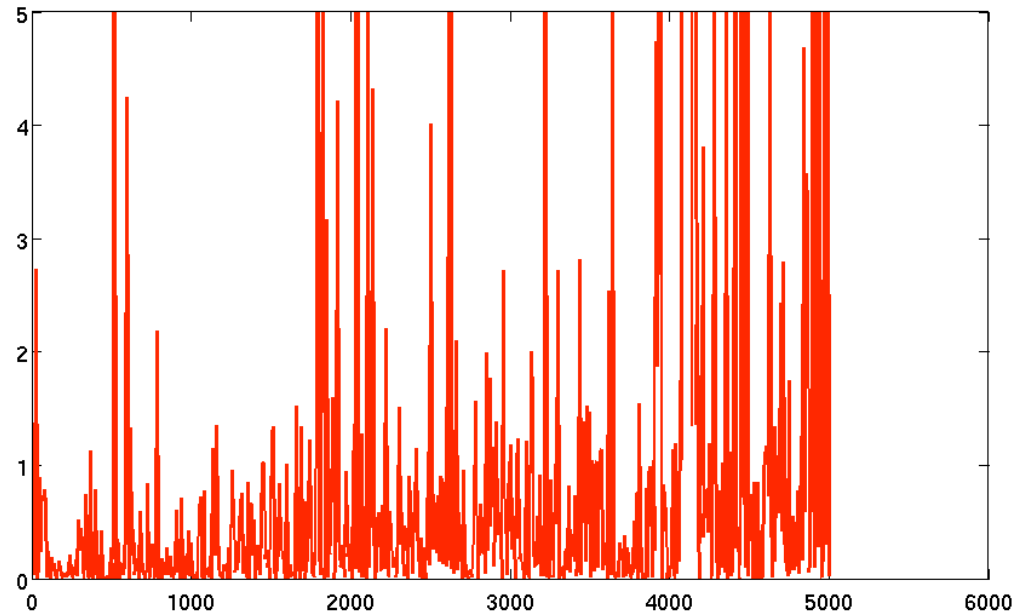
DGFI: 16

Combination: weighting



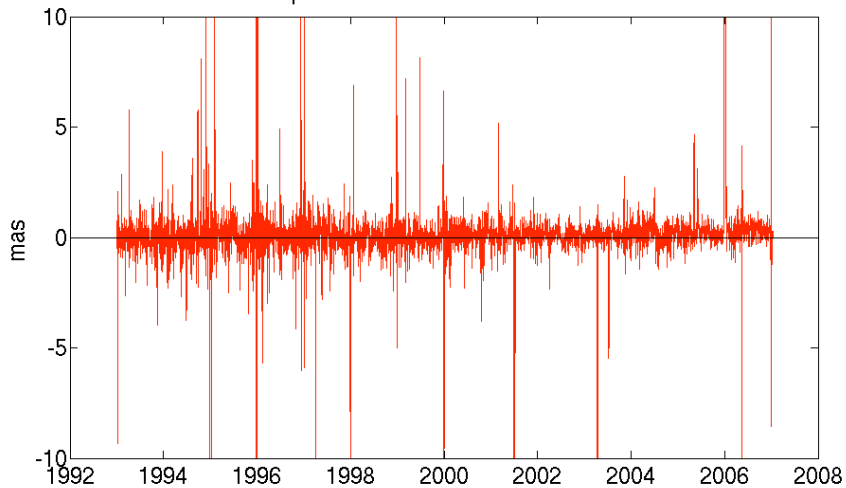
Mean diagonal elements for
coordinates:
DGFI divided by GFZ ~ 1

Weekly weighting factor

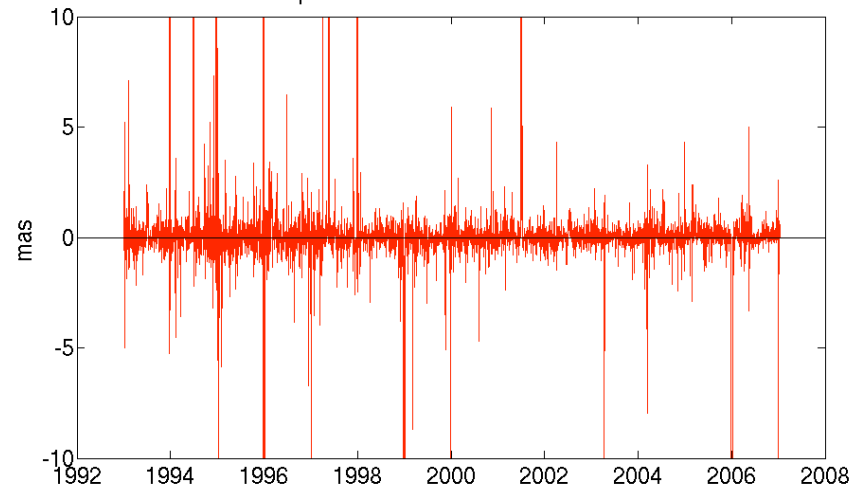


Combination: ERP w.r.t. C04

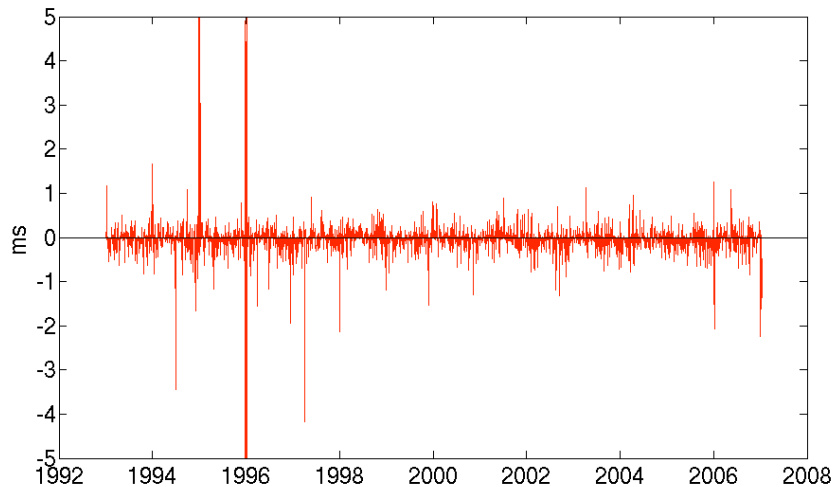
X pole: GFZ & DGFI combined



Y pole: GFZ & DGFI combined



UT1: GFZ & DGFI combined



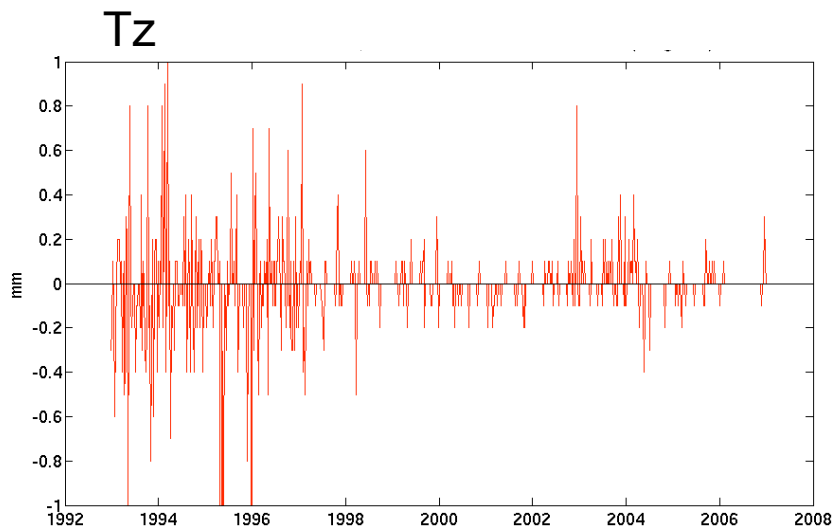
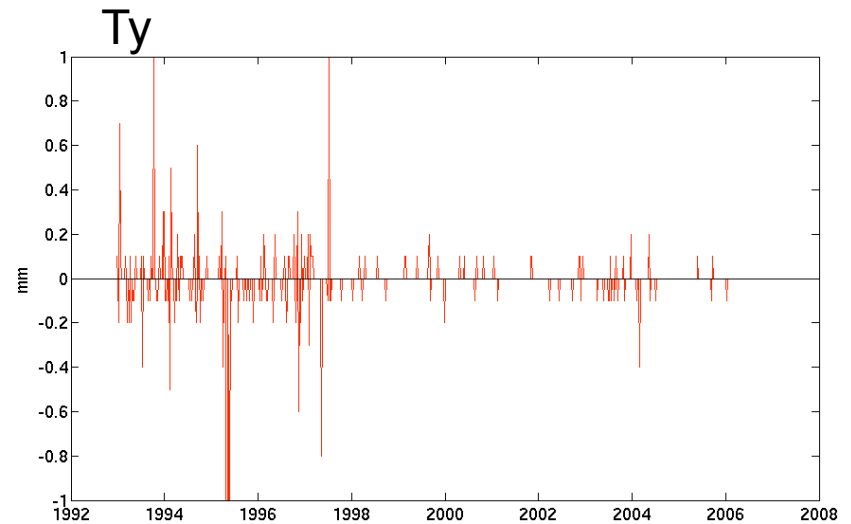
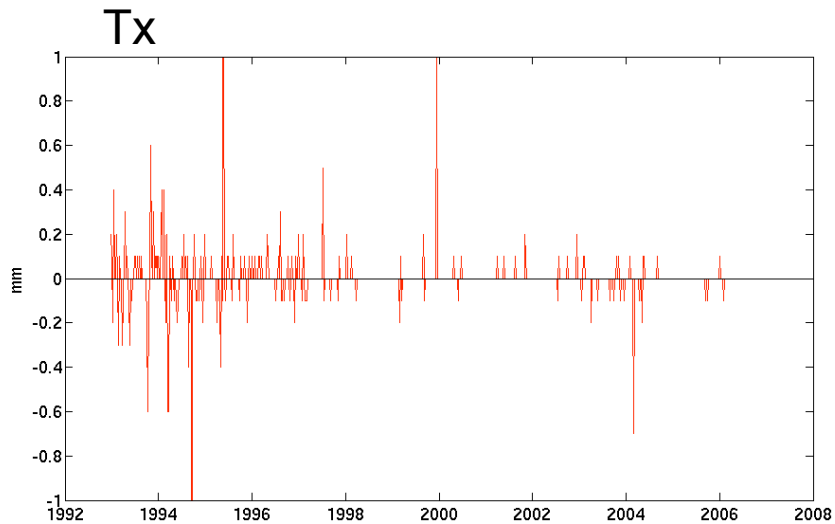
WRMS (outliers removed)

X 0.34 mas

Y 0.32 mas

UT1 0.00015 ms

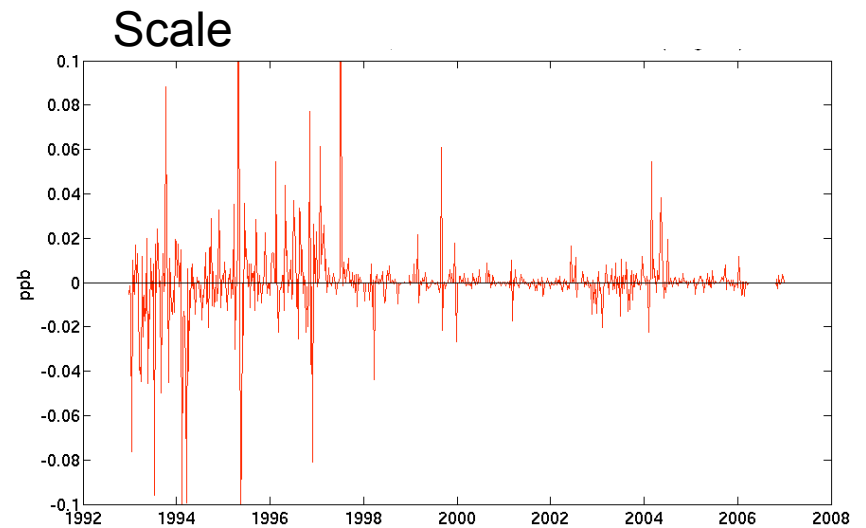
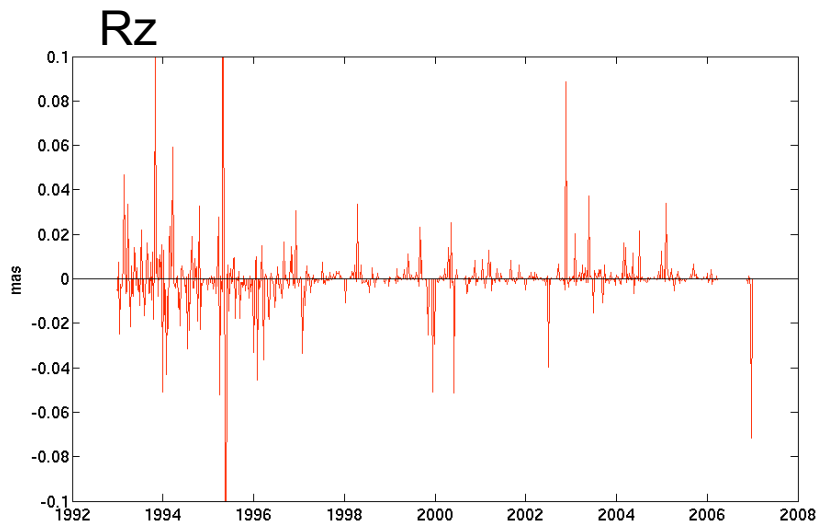
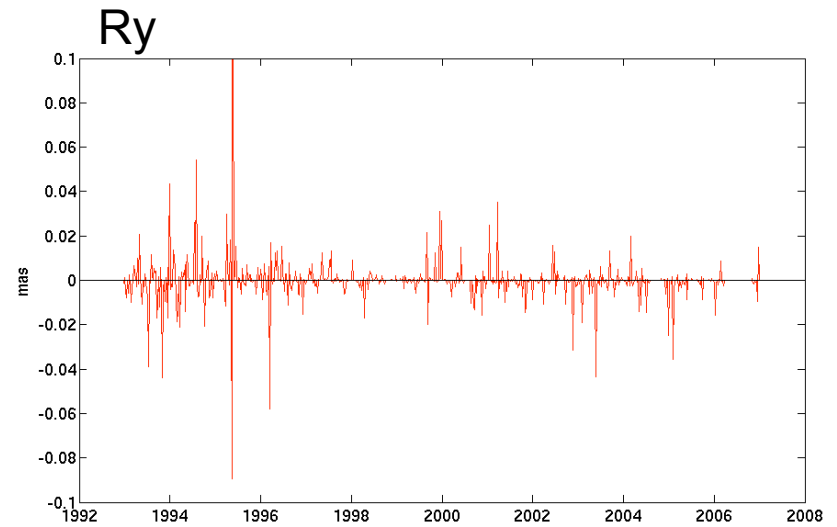
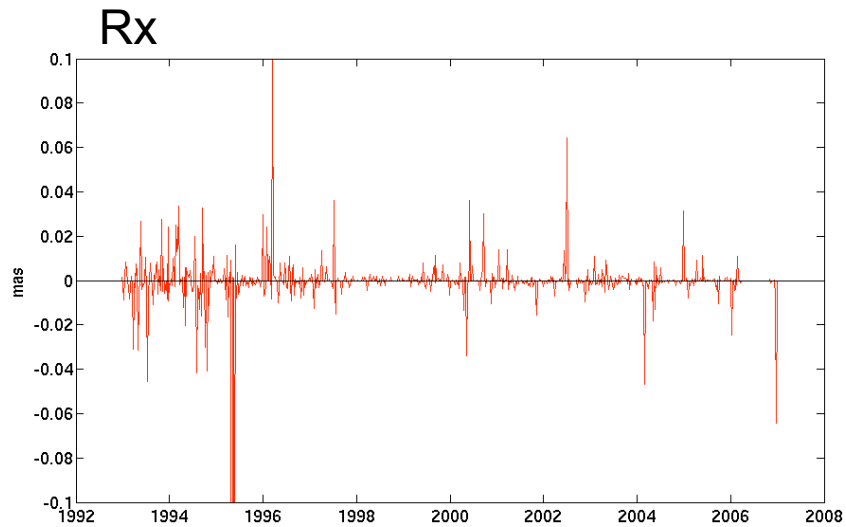
Combination: Range Biases stacked



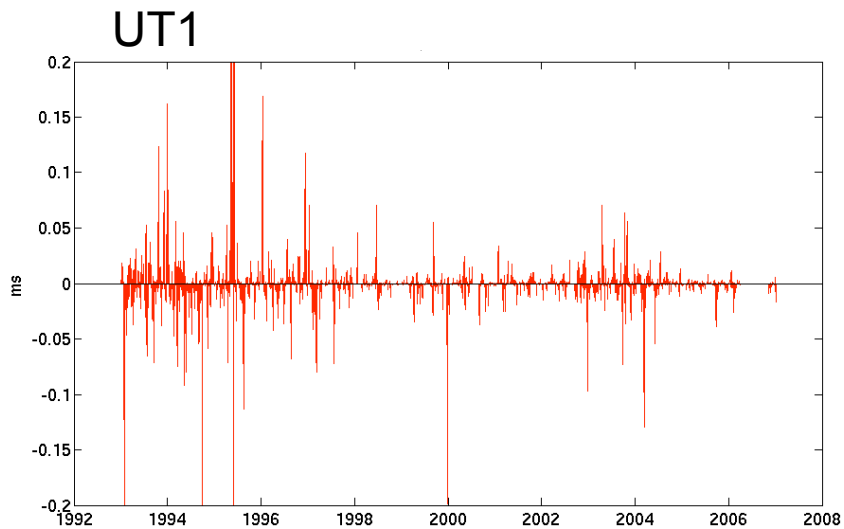
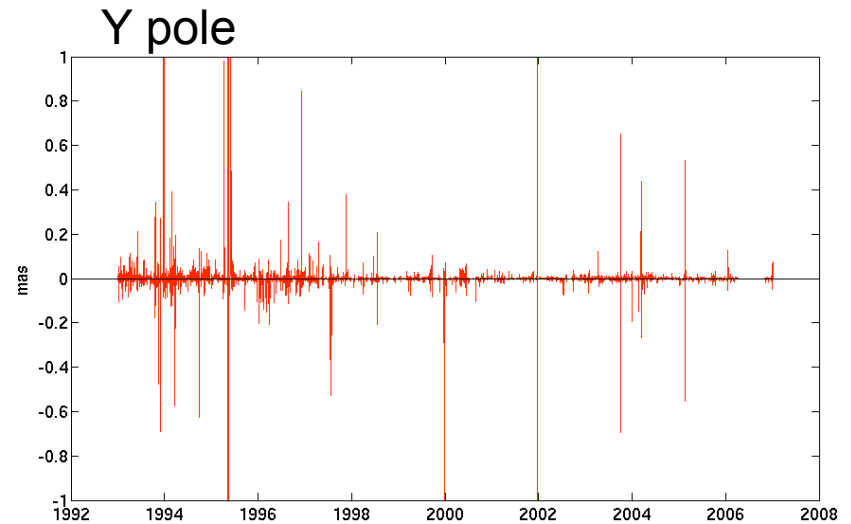
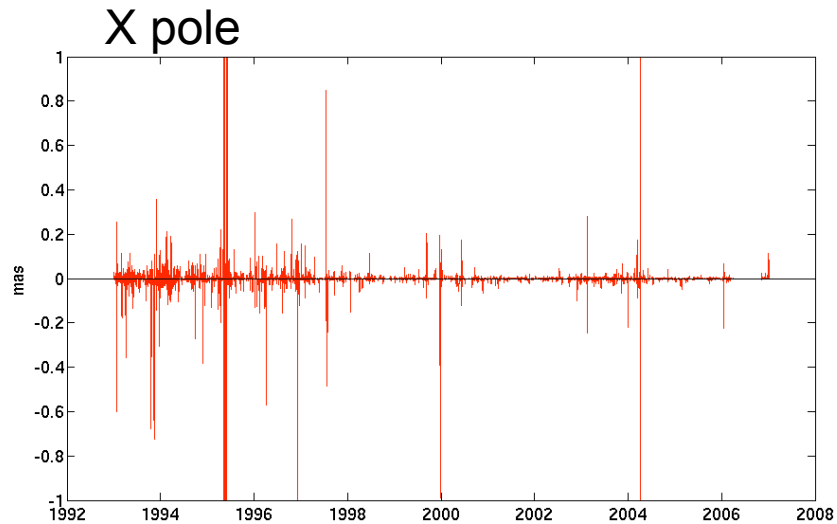
Helmert parameters:

Weekly solutions with RB
combined w.r.t. weekly
solutions with RB not
combined

Combination: Range Biases stacked



Combination: Range Biases stacked



ERP differences:

Weekly solutions with RB
combined w.r.t. weekly
solutions with RB not
combined

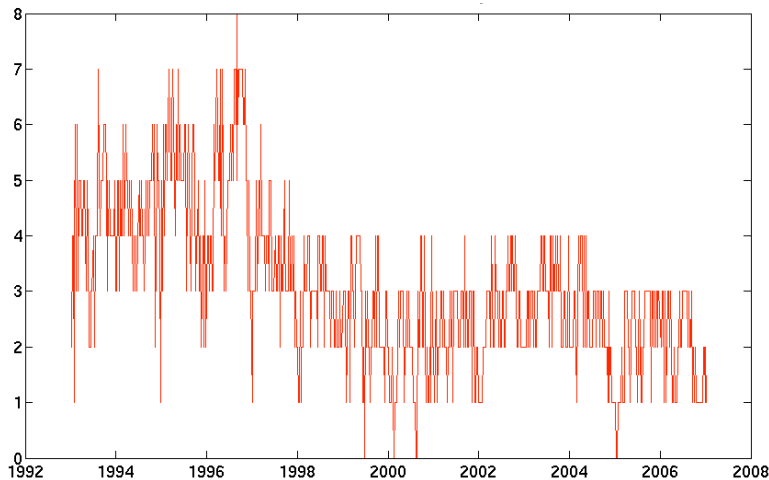
Influence of Range Biases

GFZ GGOS-D weekly test solutions:

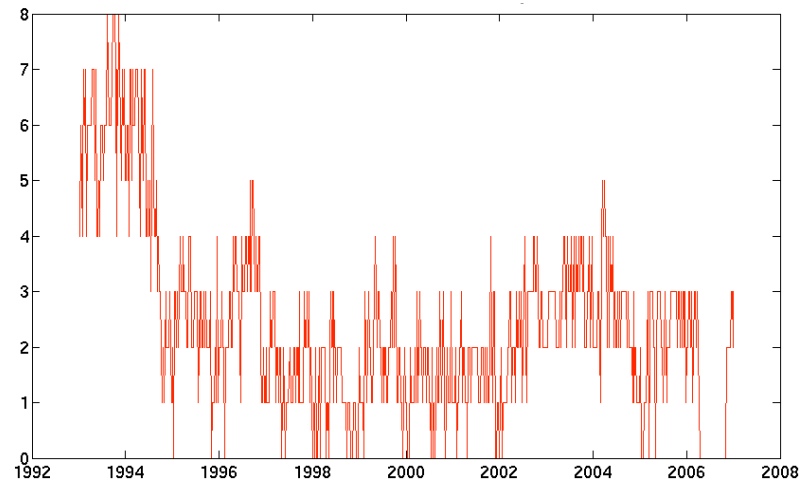
1. Range Biases estimated
2. Range Biases fixed to zero

Number of range biases:

GFZ

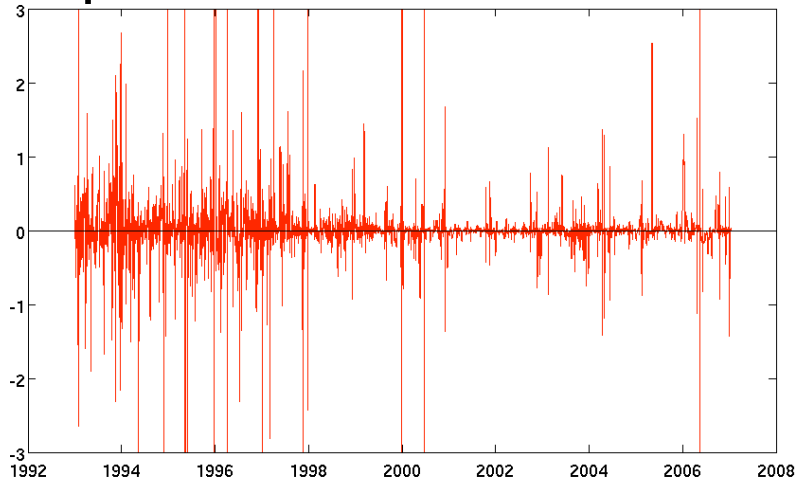


DGFI

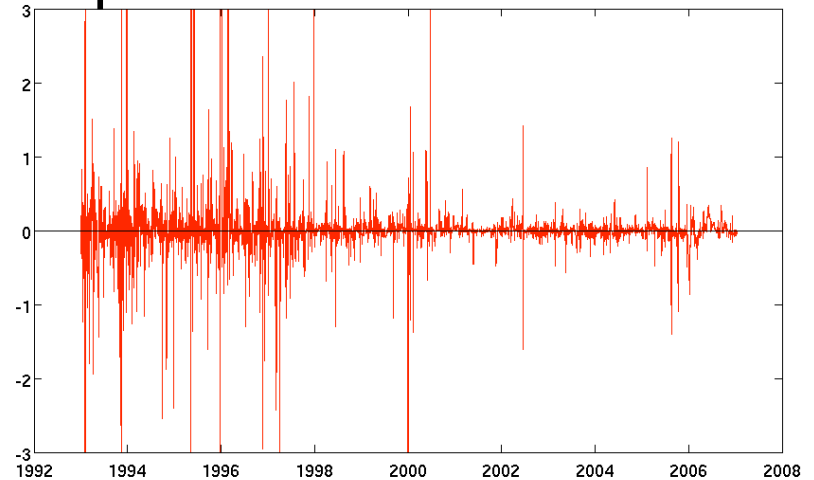


Influence of Range Biases: ERP

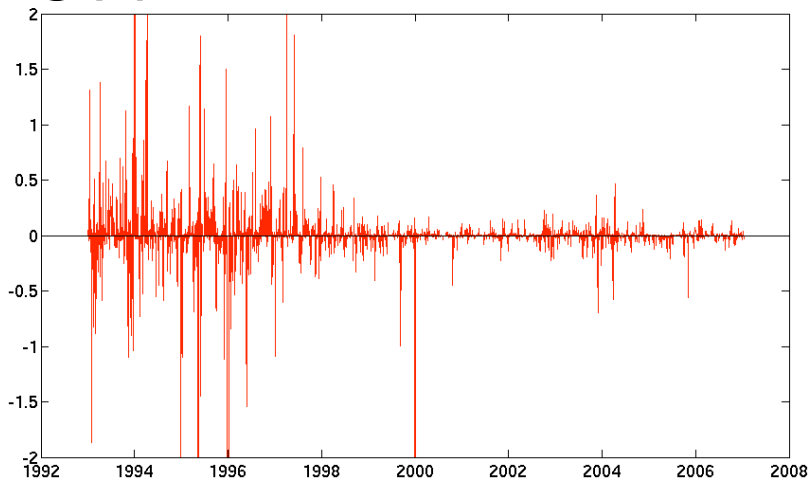
X pole



Y pole



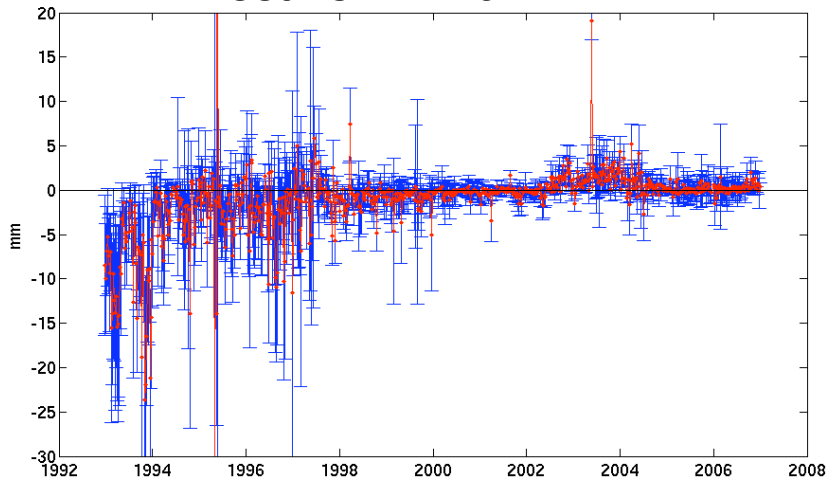
UT1



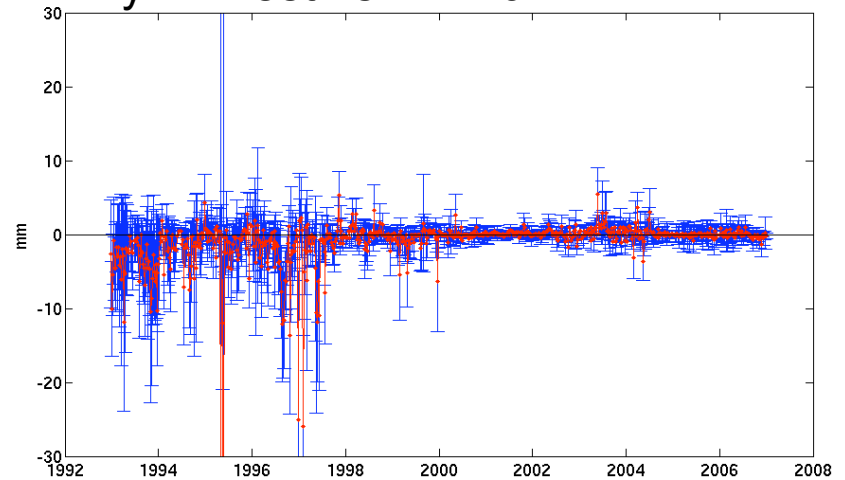
ERP differences:
GFZ solution with Range Biases estimated vs.
Range Biases fixed to zero

Influence of Range Biases: coordinates

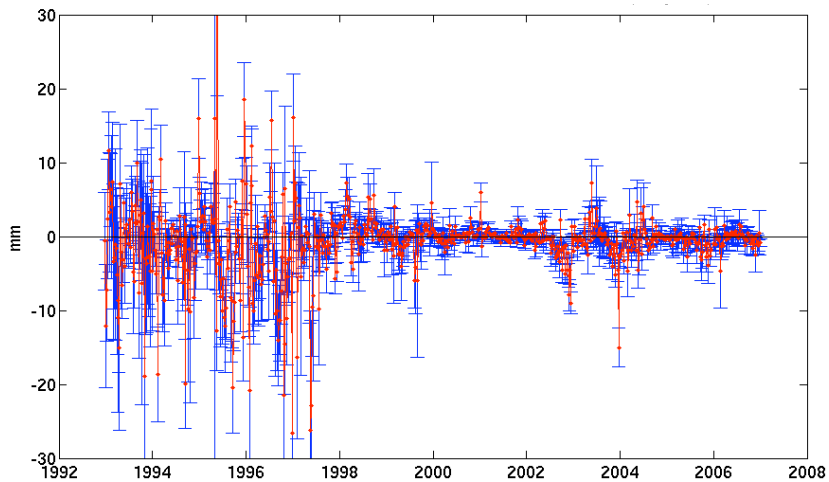
Tx: RB est vs. RB=0



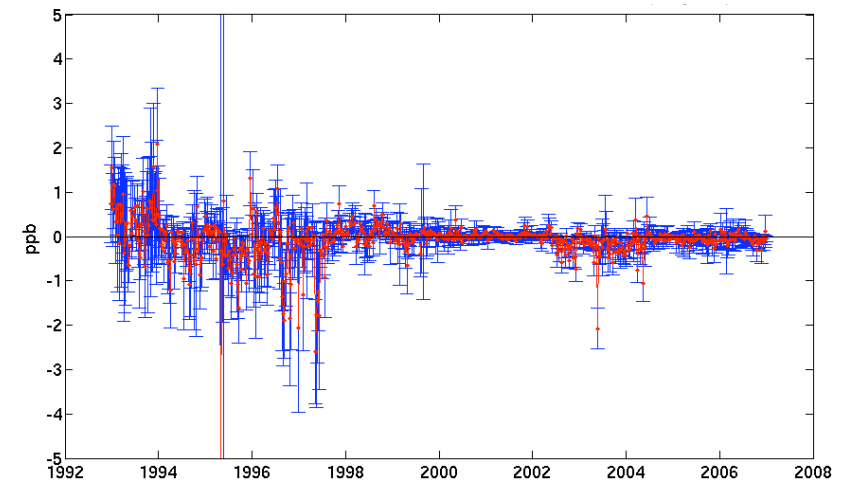
Ty: RB est vs. RB=0



Tz: RB est vs. RB=0

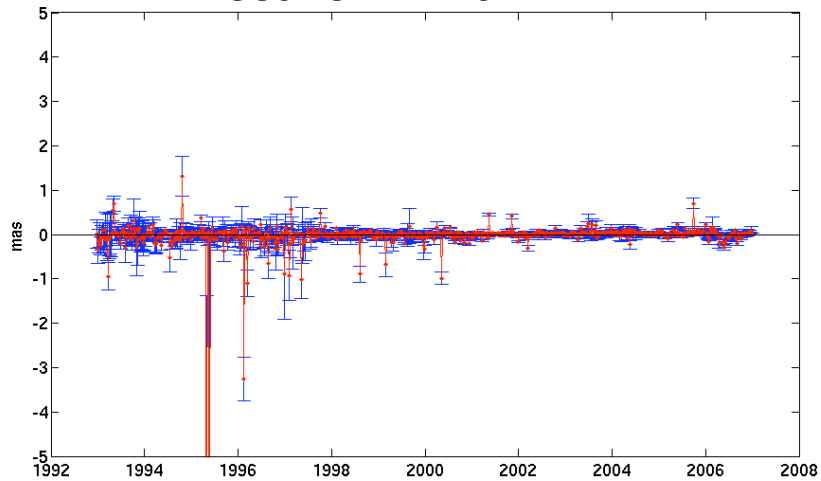


Scale: RB est vs. RB=0

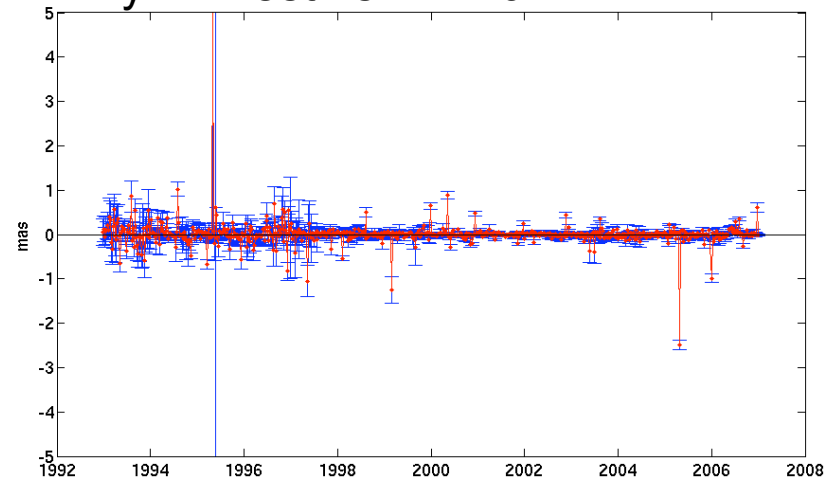


Influence of Range Biases: coordinates

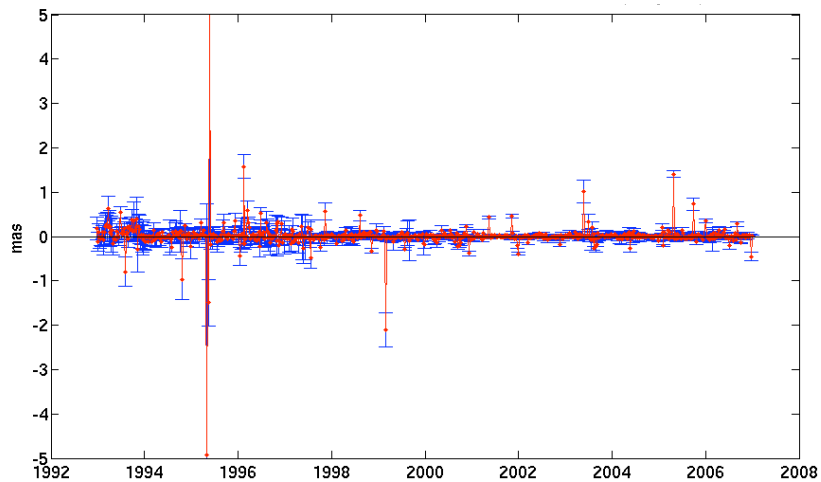
Rx: RB est vs. RB=0



Ry: RB est vs. RB=0

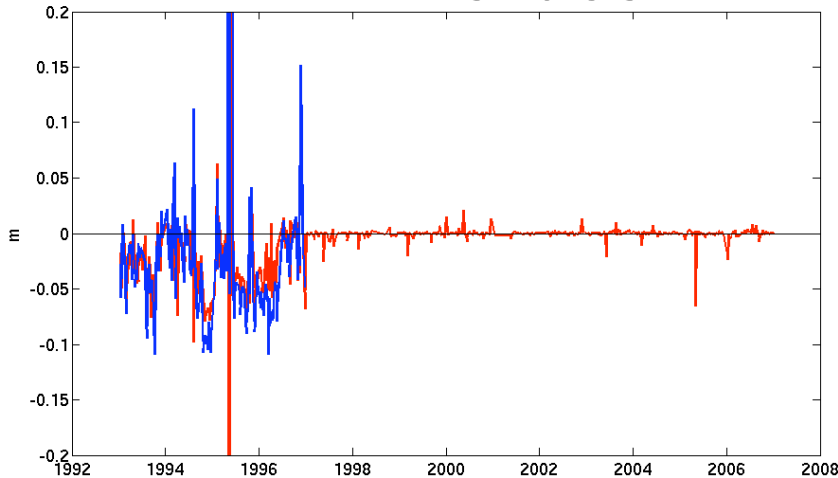


Rz: RB est vs. RB=0

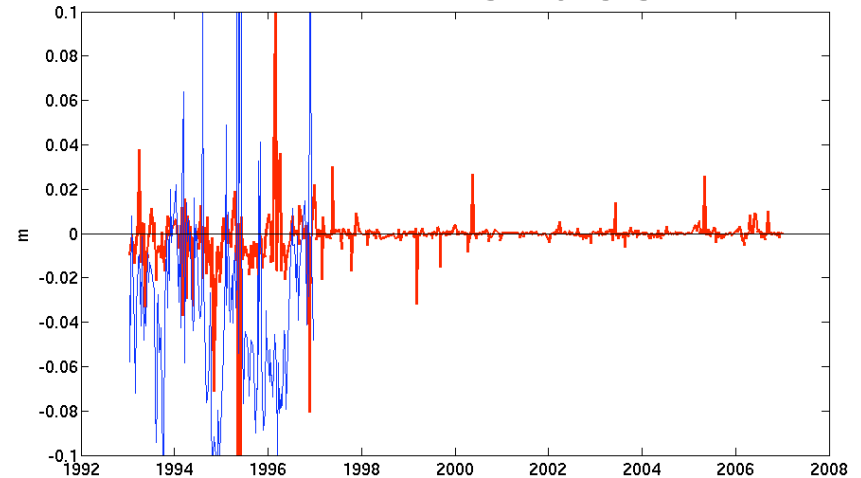


Influence of Range Biases: coordinates

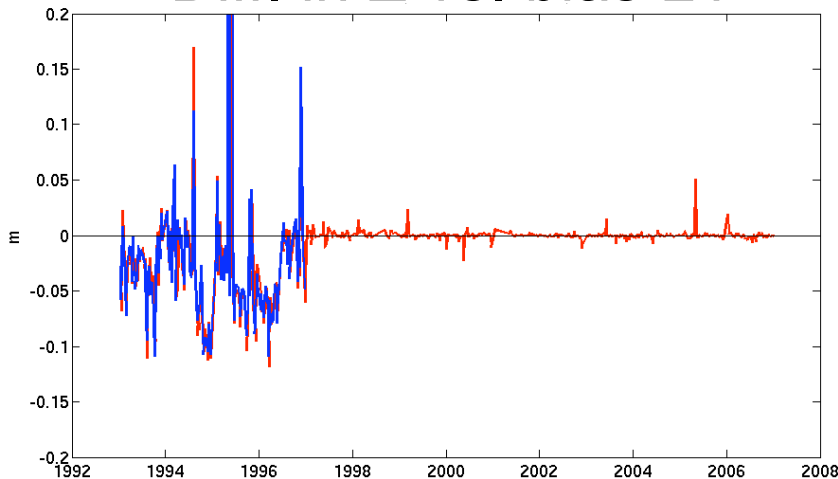
Diff. in X vs. bias L1



Diff. in Y vs. bias L1



Diff. in Z vs. bias L1

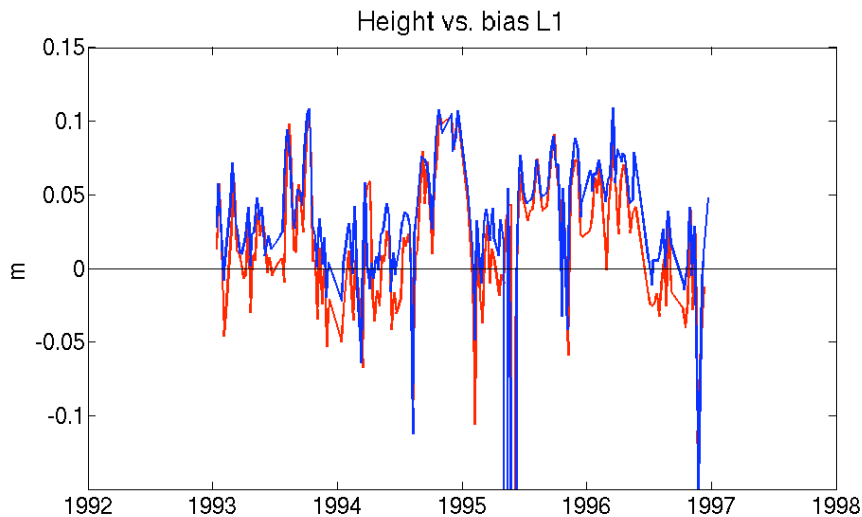
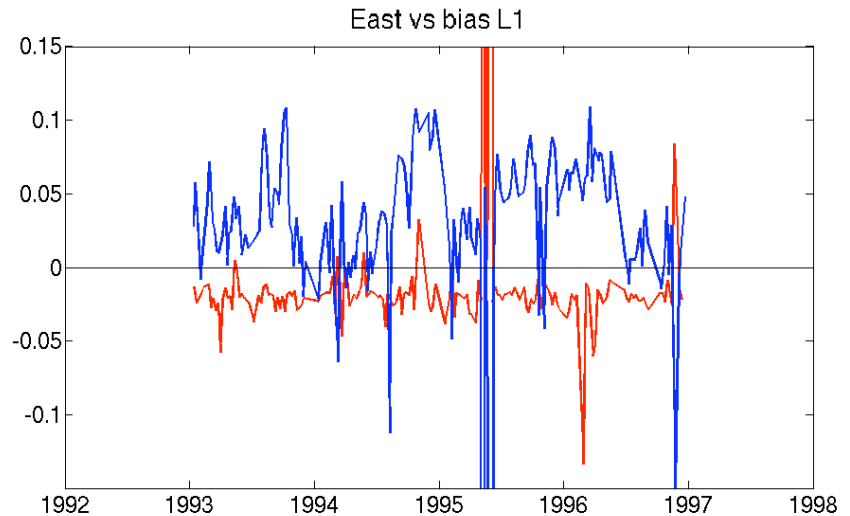
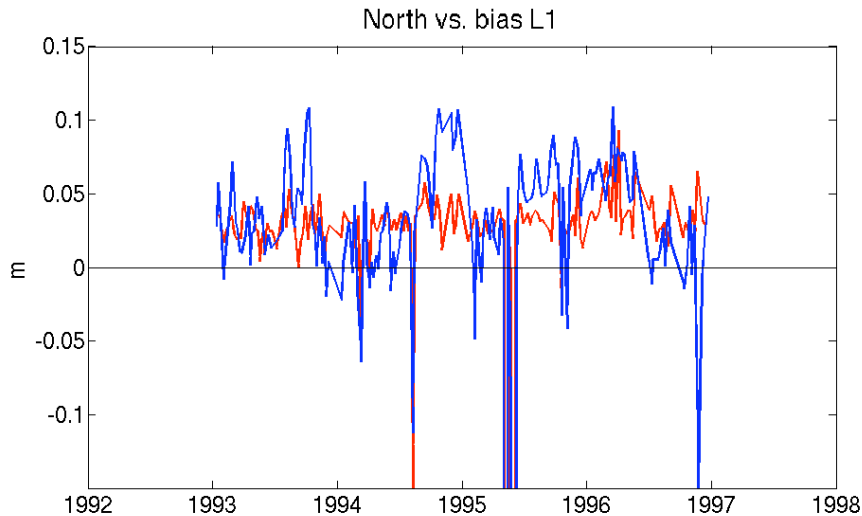


Wettzell

(x,y,z) coordinate differences:

GFZ solution with Range
Biases estimated vs. Range
Biases fixed to zero

Influence of Range Biases: coordinates

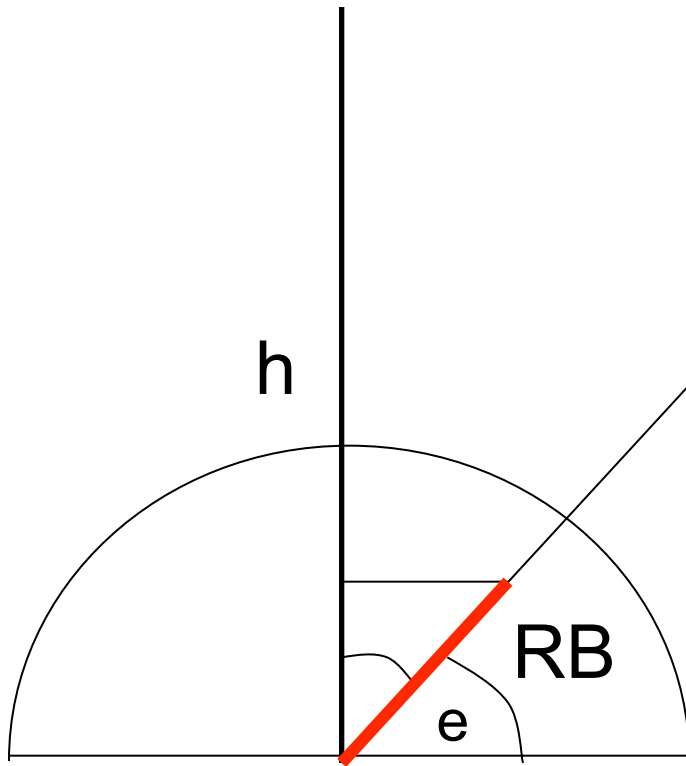


Wettzell

(e-n-h) coordinate differences:

GFZ solution with Range Biases estimated vs. Range Biases fixed to zero

Influence of Range Biases: coordinates



$$dh(e) = RB \cos(90-e)$$

$$e = 20 \text{ deg}$$

$$dh(20) = RB \cos(70)$$

$$dh(20) \sim 0.66 RB$$

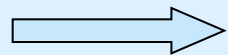
$$dh(0) = RB$$

1st degree Gravity Field Coefficients

GFZ GGOS-D weekly test solutions:

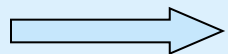
1. 1st degree GFC estimated
2. 1st degree GFC fixed to zero

- 1st degree Gravity Field Coefficients are fixed (to zero)



No-Net-Rotation + 1 UT1 fixed

- Estimation of 1st degree Gravity Field Coefficients

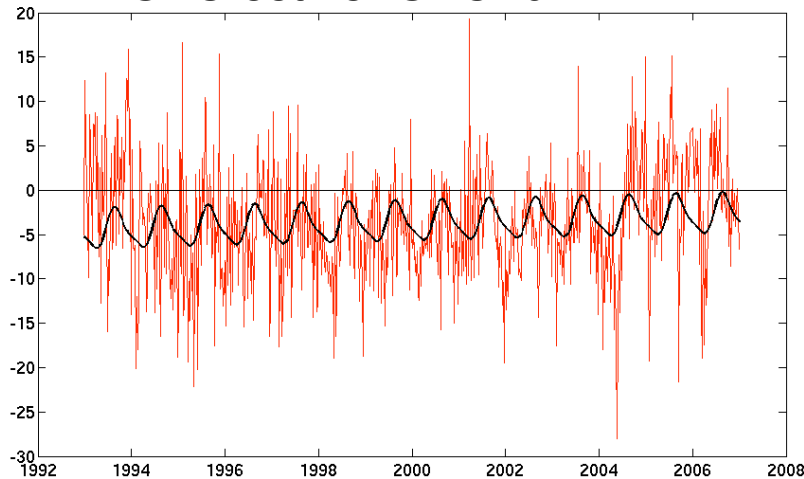


No-Net-Rotation + No-Net-Translation

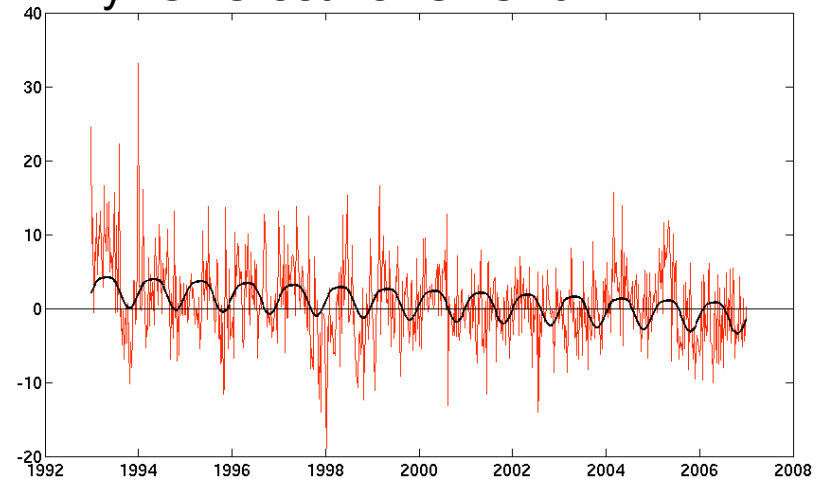
+ 1 UT1 fixed

1st degree Gravity Field Coefficients

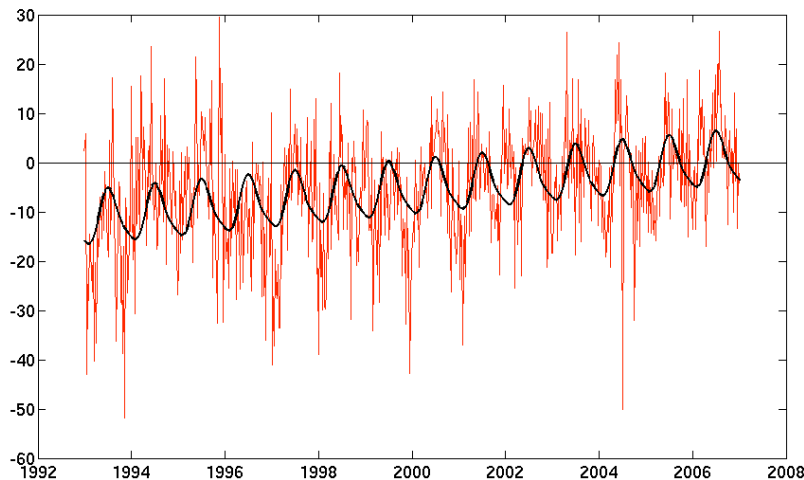
Tx: GFC est vs. GFC=0



Ty: GFC est vs. GFC=0



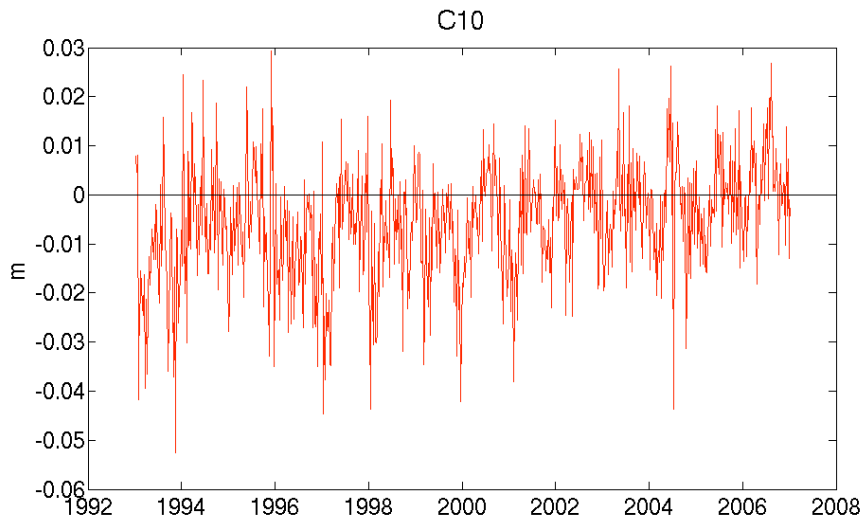
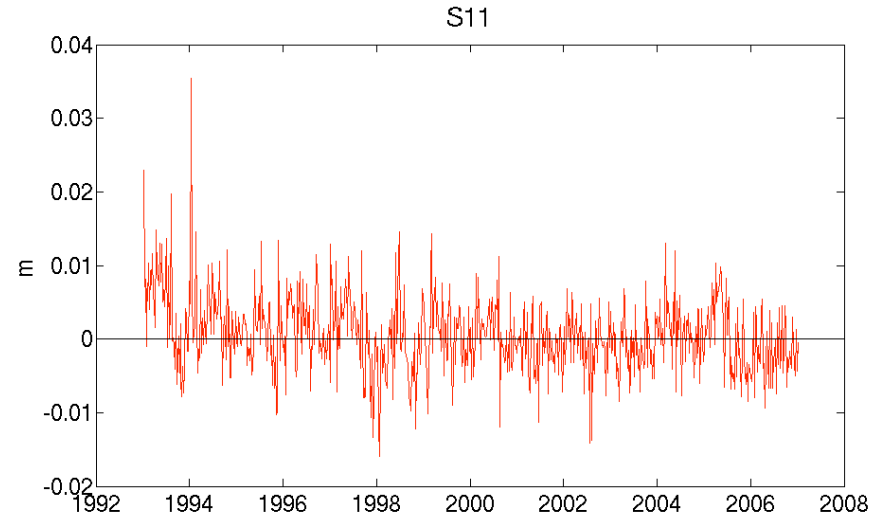
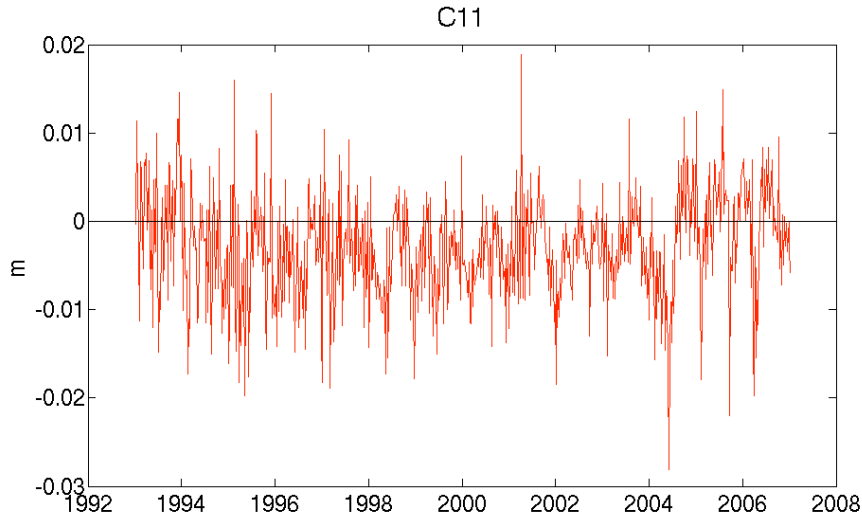
Tz: GFC est vs. GFC=0



Helmert parameters:

1st deg GFC estimated
(NNR-NNT) w.r.t. 1st deg
GFC fixed to zero (NNR)

1st degree Gravity Field Coefficients



1st degree
Gravity Field Coefficients:

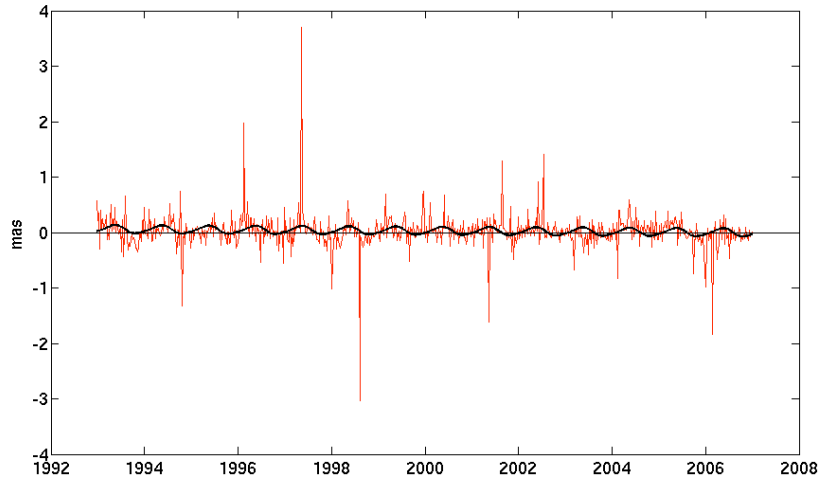
C11 \longleftrightarrow X

S11 \longleftrightarrow Y

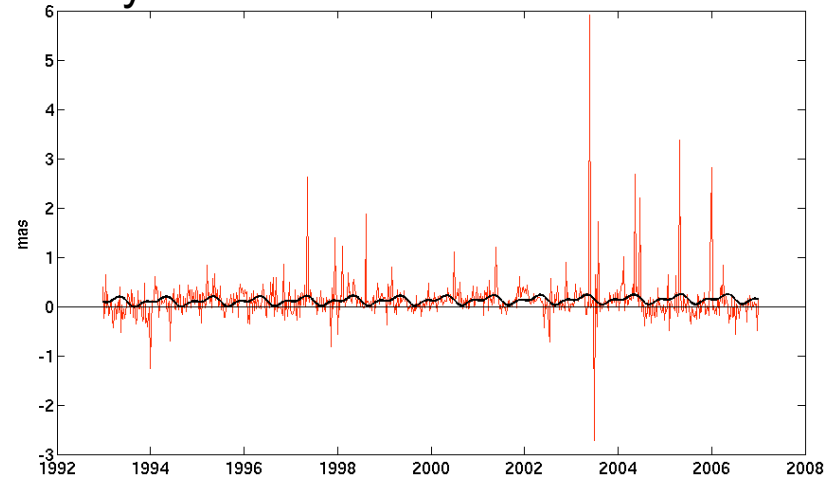
C10 \longleftrightarrow Z

1st degree Gravity Field Coefficients

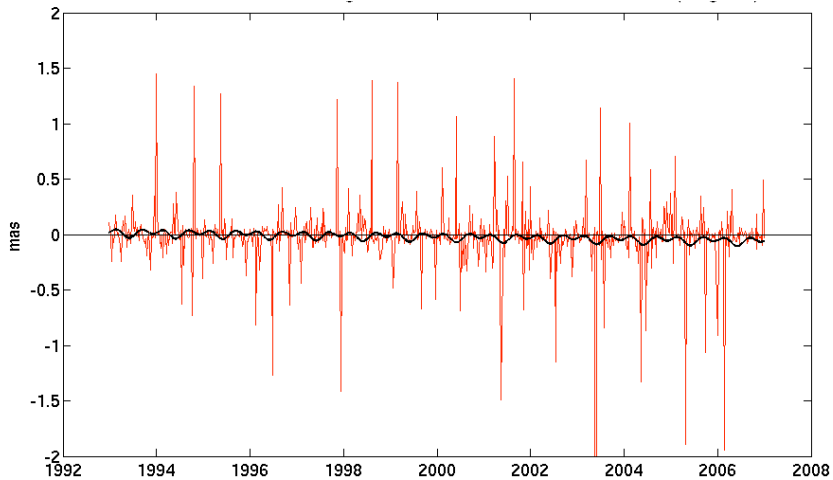
Rx: GFC est vs. GFC=0



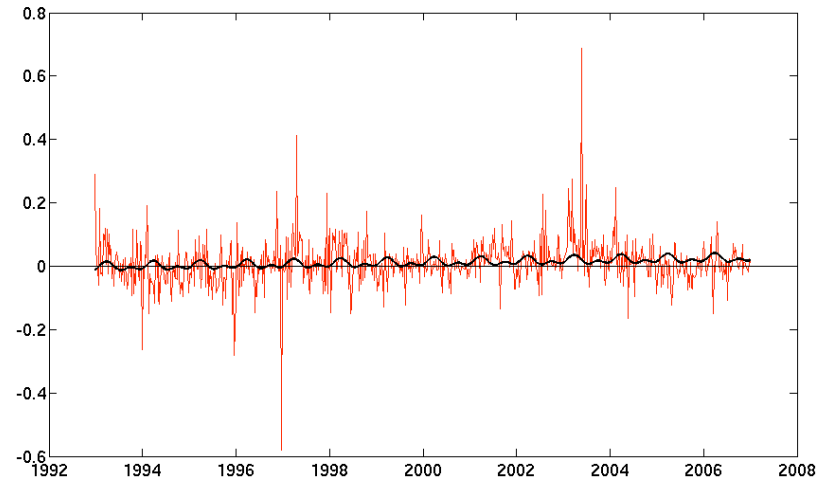
Ry: GFC est vs. GFC=0



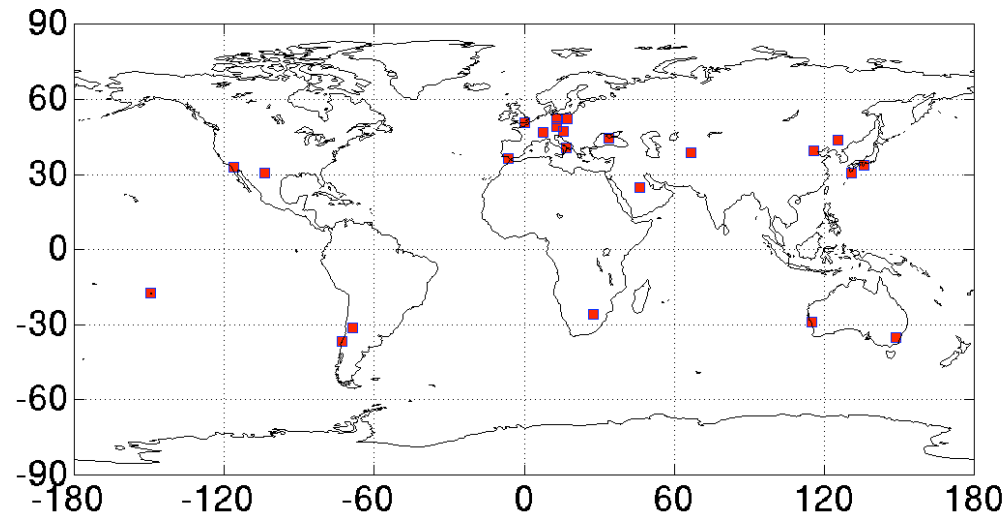
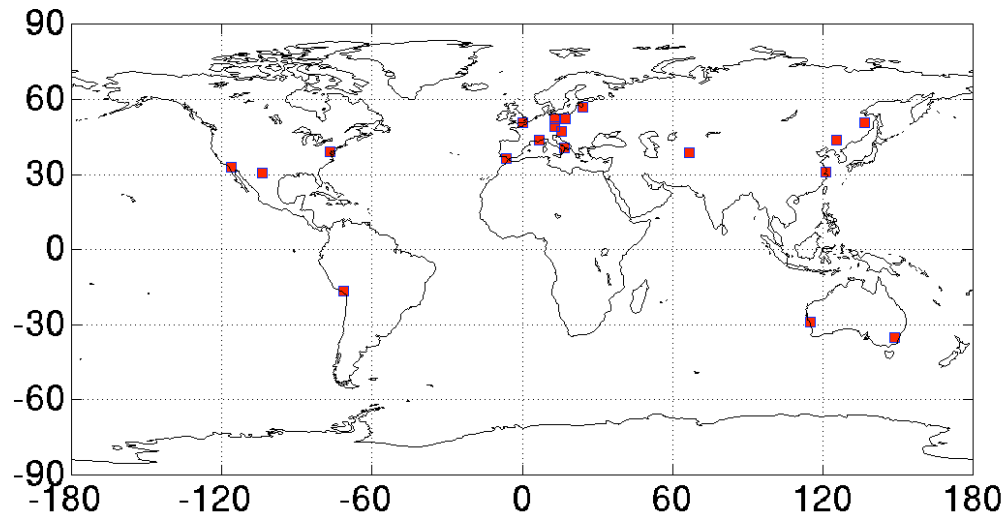
Rz: GFC est vs. GFC=0



Scale: GFC est vs. GFC=0



Network geometry



Network geometry: correlations

Helmert parameters:

$$\vec{X}_{\text{est}} = \mu \cdot \mathbf{R} \cdot \vec{X}_{\text{ref}} + \vec{T}$$

μ scale factor

\mathbf{R} matrix of rotations

\mathbf{T} translations

Design matrix:

$$\mathbf{A} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 0 & 0 & x_i & 0 & z_i & -y_i \\ 0 & 1 & 0 & y_i & -z_i & 0 & x_i \\ 0 & 0 & 1 & z_i & y_i & -x_i & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

Normal equation matrix:

$$\begin{pmatrix} N & 0 & 0 & \sum x_i & 0 & \sum z_i & -\sum y_i \\ 0 & N & 0 & \sum y_i & -\sum z_i & 0 & \sum x_i \\ 0 & 0 & N & \sum z_i & \sum y_i & -\sum x_i & 0 \\ \sum x_i & \sum y_i & \sum z_i & \sum r_i^2 & 0 & 0 & 0 \\ 0 & -\sum z_i & -\sum y_i & 0 & \sum z_i^2 + y_i^2 & -\sum x_i y_i & -\sum x_i z_i \\ \sum z_i & 0 & -\sum x_i & 0 & -\sum x_i y_i & \sum z_i^2 + x_i^2 & -\sum y_i z_i \\ -\sum y_i & \sum x_i & 0 & 0 & -\sum x_i z_i & -\sum y_i z_i & \sum y_i^2 + x_i^2 \end{pmatrix}$$

Network geometry: correlations

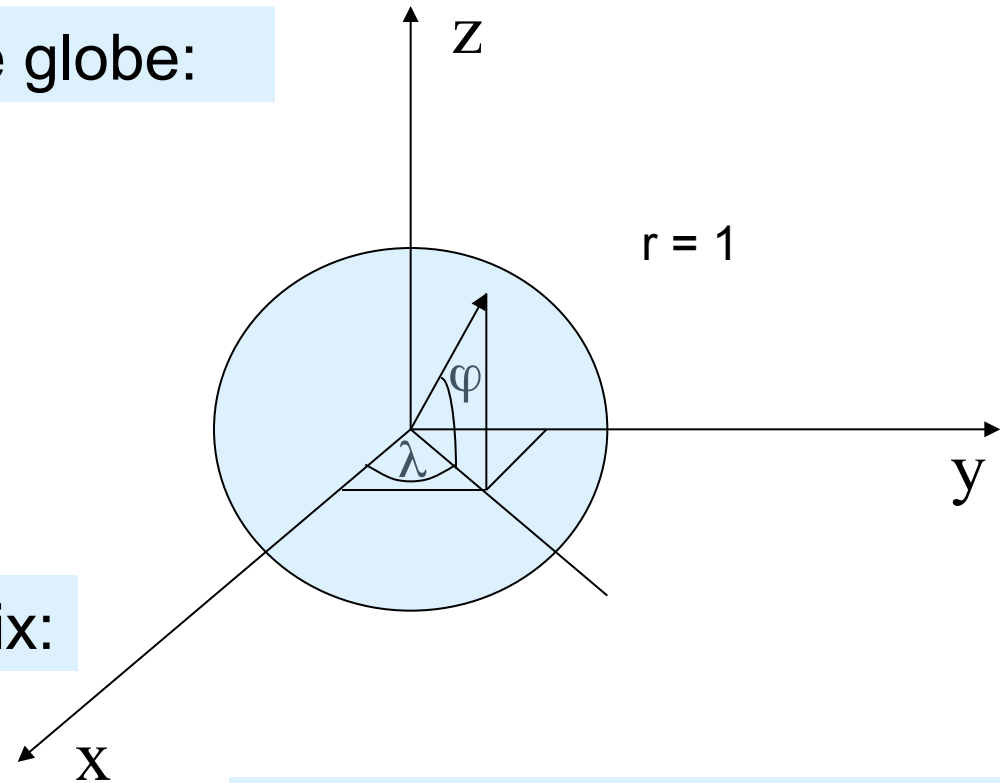
Uniform distribution over the globe:

$$\int_{-\pi/2}^{\pi/2} \int_0^{2\pi} d\varphi d\lambda \cos \varphi$$

$$x = \cos \varphi \cos \lambda$$

$$y = \cos \varphi \sin \lambda$$

$$z = \sin \varphi \longrightarrow 2\pi \frac{\sin^2 \varphi}{2} \Bigg|_{\varphi_1}^{\varphi_2}$$

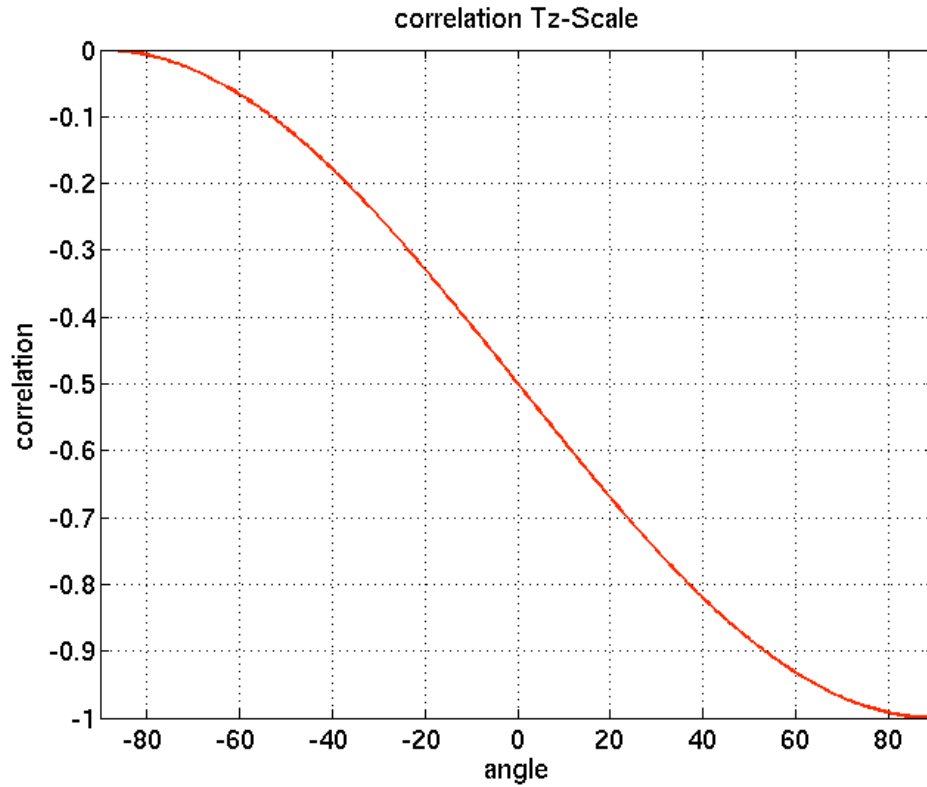


Covariance/correlation matrix:

	Tx	Ty	Tz	Sc	Rx	Ry	Rz
Tx	A	0	0	0	0	B	0
Ty	0	A	0	0	-B	0	0
Tz	0	0	A	B	0	0	0
Sc	0	0	B	A	0	0	0
Rx	0	-B	0	0	A ₁	0	0
Ry	B	0	0	0	0	A ₂	0
Rz	0	0	0	0	0	0	A ₃

Asymmetric distribution over latitude cause correlations Tx - Ry, Ty - Rx and Tz - Scale

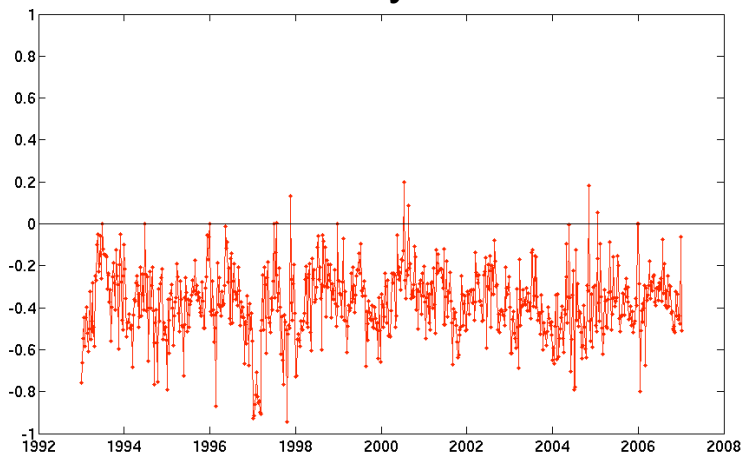
Network geometry: correlations



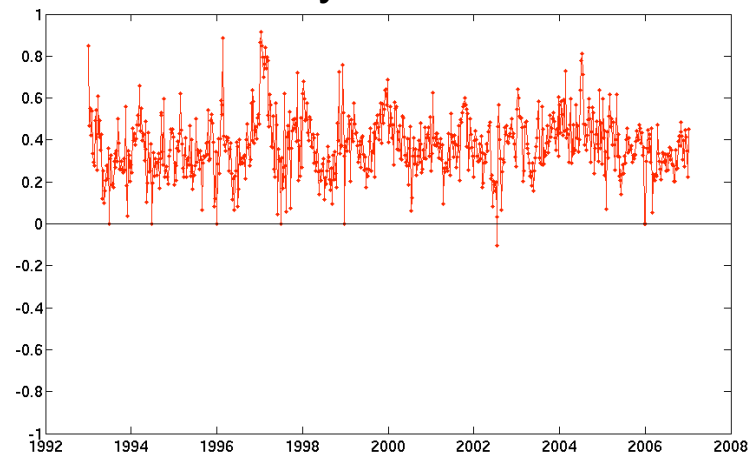
$$-\frac{1}{2}(\sin \varphi + 1)$$

Network geometry: correlations

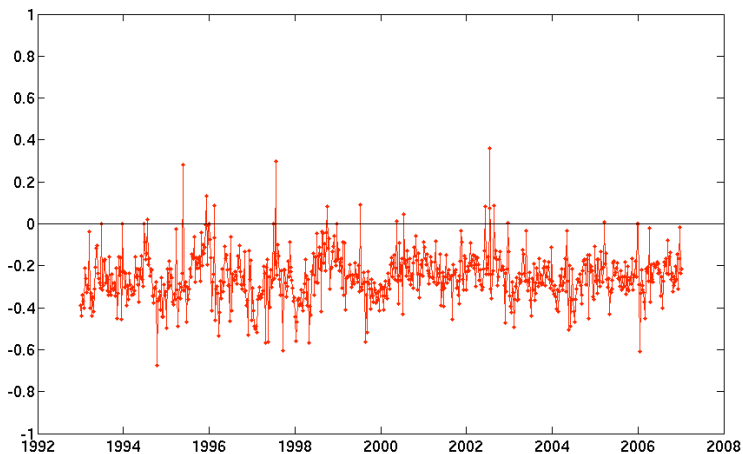
Tx-Ry



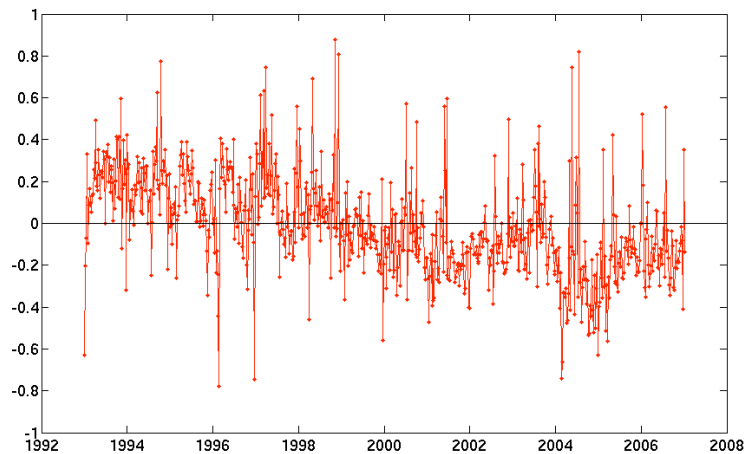
Ty-Rx



Tz-Scale

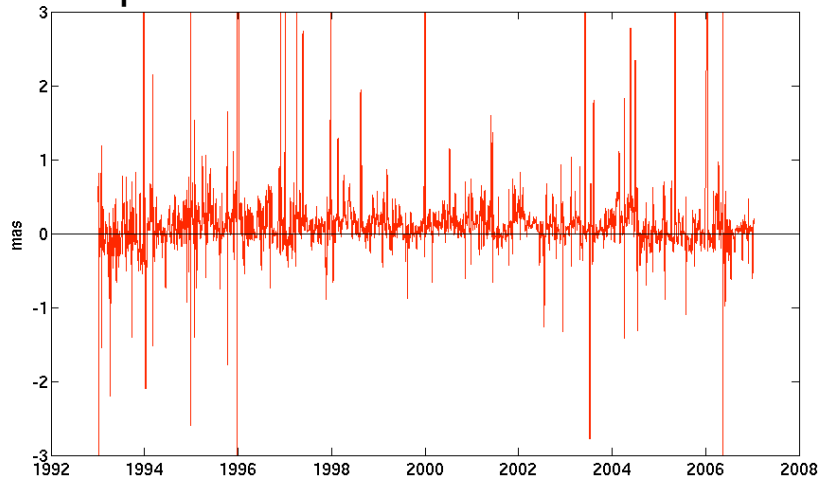


Tz-Rx

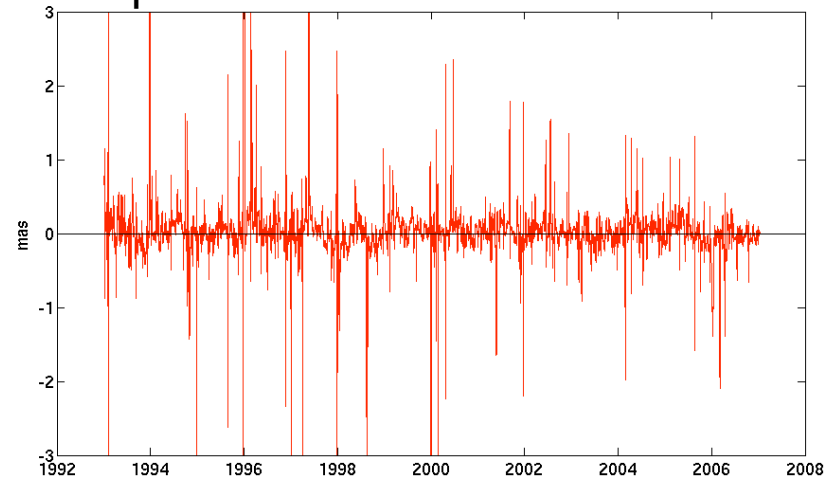


1st degree Gravity Field Coefficients: ERP

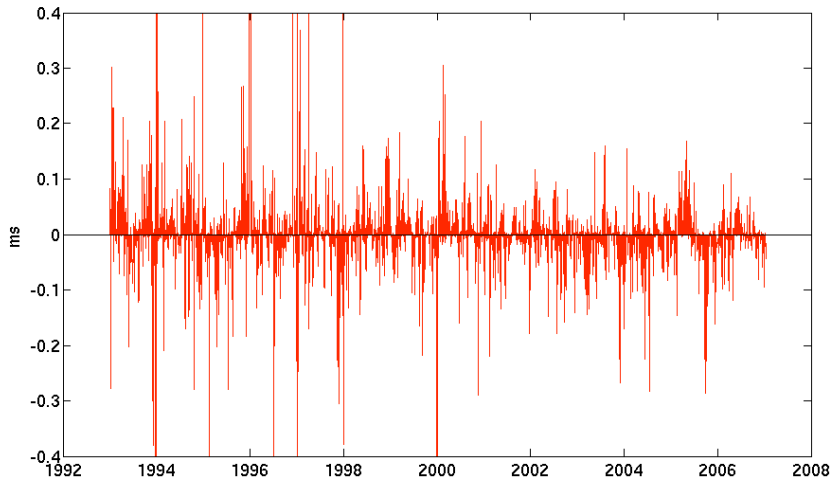
X pole



Y pole



UT1



ERP differences:
GFZ solution with GFC
estimated minus GFC
fixed to zero

Summary

Estimating 1st degree Gravity Field Coefficients vs. fixing them to zero



- systematic differences in ERP
- RMS of ERP time series remains on the same level

Different sets of stations with Range Biases estimated

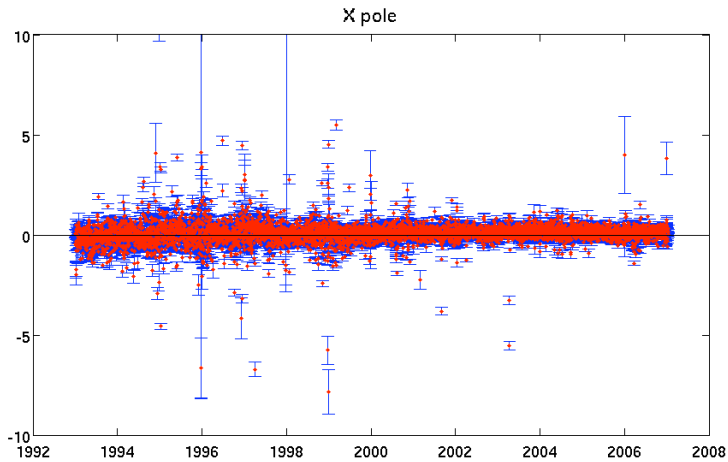
- differences in coordinates of stations and ERP

Combination of Range Biases

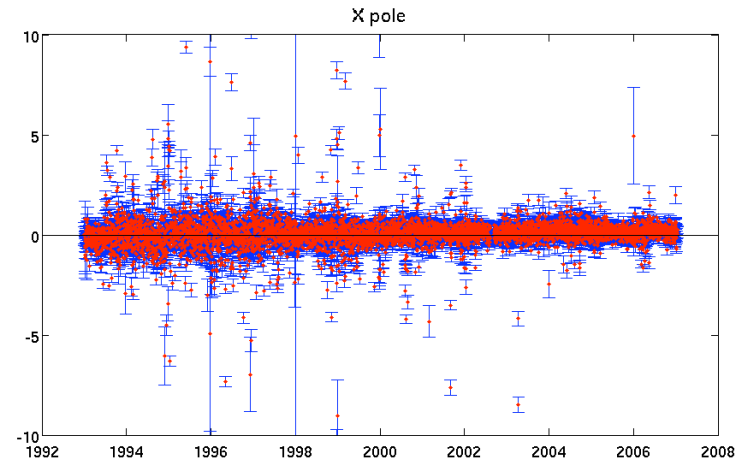
- differences in coordinates of stations and ERP, but not significant

Multiyear solution: X pole

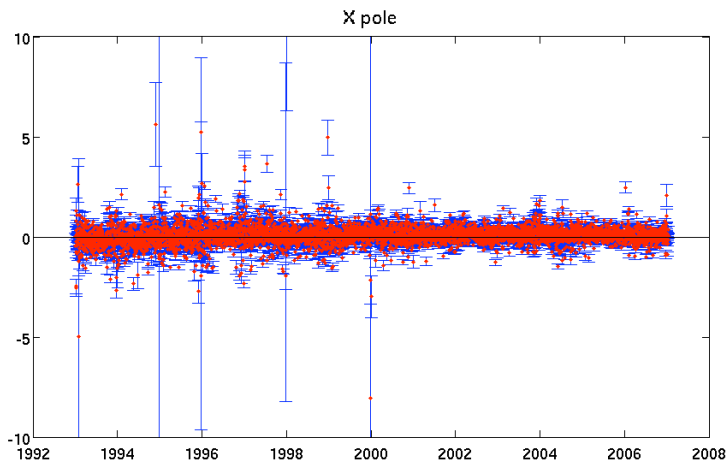
combined



DGFI



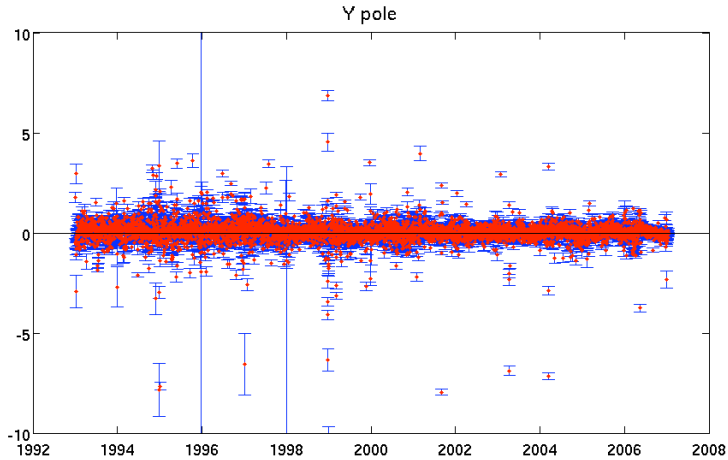
GFZ



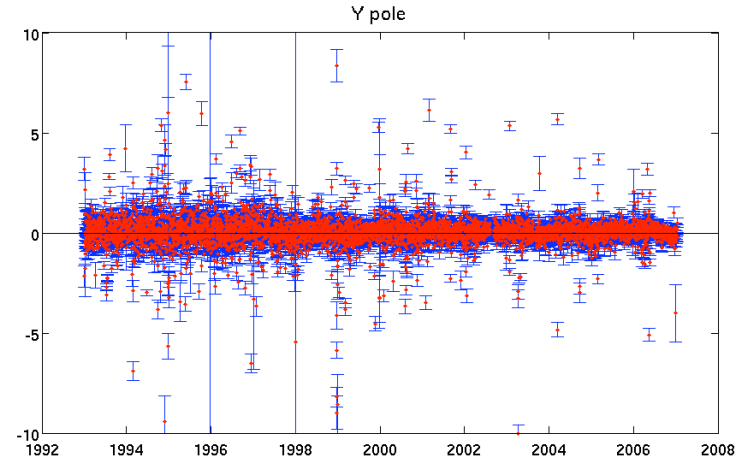
	RMS	WRMS
comb	0.9	0.36
DGFI	1.6	0.58
GFZ	0.7	0.31

Multiyear solution: Y pole

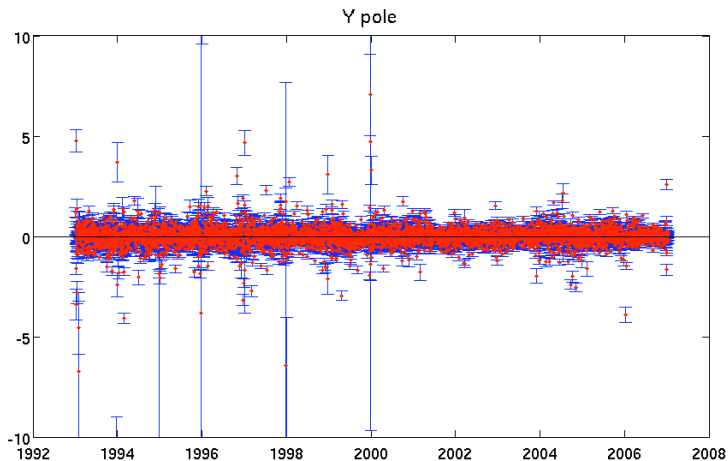
combined



DGFI



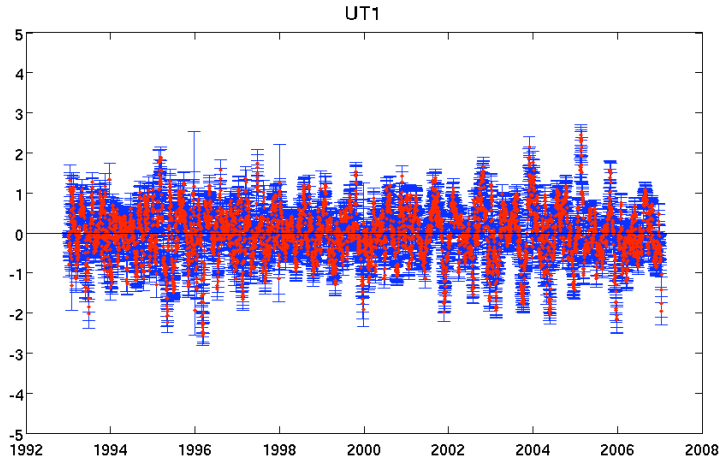
GFZ



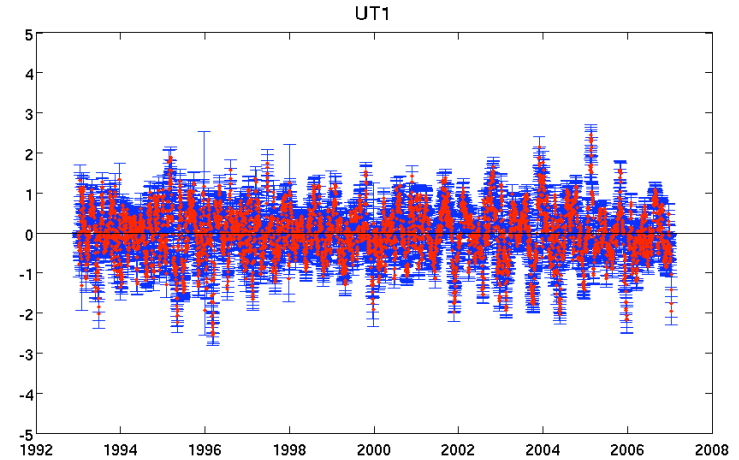
	RMS	WRMS
comb	1.0	0.46
DGFI	1.8	0.76
GFZ	1.2	0.32

Multiyear solution: UT1

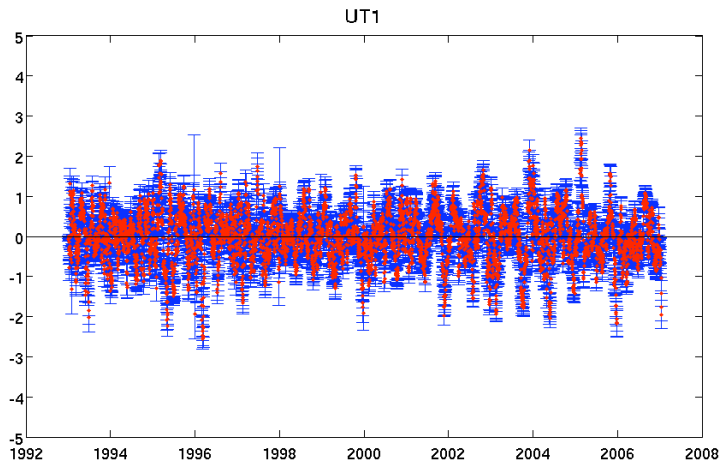
combined



DGFI



GFZ



	RMS	WRMS
comb	0.42	0.00027
DGFI	0.71	0.00032
GFZ	0.64	0.00062

Thank you!